

# History of differential geometry

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## General scheme and topics

We deal with 5 topics, one topic each day (*cum grano salis*). I plan to give 2 hours of lectures (45 min each) each day. After a break we continue with working sessions in which selected parts of sources or specific historical questions are discussed.

Below you find the topics with indication of the main (historical) literature. Not all of it is easily accessible; nevertheless the list may help those of the participants who want to go deeper in any of the points presented in the course. The references are grouped into historical literature, selected sources, and some exemplary cases of mathematical literature which can be consulted. There are many good books on (the mathematics of) differential geometry. You can take essentially any of them. In our context, (Spivak 1970) is of particular value; it contains a detailed mathematical discussion close to the historical material in Gauss and Riemann (and an English translation of Riemann's inaugural lecture).

## 1. Outline and origins of curve and surface theory

- Introduction to the history of differential geometry  
(Struik 1933, Loria 1942–45, Gray 1979, Gray 1999, Boi e.a. 1992, Libermann 1978, Laptev/Rozenfeld 1996, James 1999) and (Stillwell 1989, chap. 16)
- Curvature concepts before Gauss  
(Gericke 1982, Struik 1933)
- Gauss' theory of surfaces  
(Dombrowski 1979, Reich 1973, Nabonnand 1995)
- Gaussian surface theory and foundations of physical geometry  
(Miller 1972, Breitenberger 1984, Bottazzini 1994, Scholz 2004b)

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## **2. Riemann's "Hypothesen"**

- origins of  $n$ -dimensional geometrical thinking  
(Scholz 1999*a*)
- Riemann and Herbart  
(Scholz 1982)
- Riemann's inaugural address of 1854  
(Portnoy 1982, Spivak 1970, Scholz 1992)
- Riemann on physical geometry and the Paris "prize essay"  
(Bottazzini/Tazzioli 1995, Bottazzini 1994, Farwell/Knee 1990, Zund 1983)

## **3. From Riemann to Ricci and Levi-Civita**

- Beltrami  
(Tazzioli 1993, Tazzioli 1997)
- Differential geometry and analytical mechanics: Liouville, Lipschitz and others.  
(Lützen 1989, Lützen 1990, Lützen 1995, Tazzioli 1994)
- The rise of vectors and tensors in differential geometry  
(Butzer 1981, Katz 1981, Katz 1999*a*, Bottazzini 1999, Reich 1989, Reich 1994, Struik 1995)
- Excursus: F. Klein's "Erlanger Programm"  
(Hawkins 1984, Boi 1990)

## **4. The concept of manifold**

- Manifolds before Poincaré  
(Scholz 1999*a*)
- Poincaré and the constructive approach to manifolds  
(Sarkaria 1999)
- Axiomatization (from Hilbert and Hausdorff to Veblen and Whitehead)  
(Scholz 1999*a*)
- Outlook  
(James 1999)

## 5. Remarks on differential geometry in the 20th century

- The impact of general relativity: affine connections  
(Norton 1999, Bottazzini 1999, Bourguignon 1992, Reich 1992, Coxeter 1975, Struik 1989)
- The impact of general relativity: generalization of differential geometric structures  
(Struik 1984, Pizzocchero 1998, Goenner 2004, Goldstein/Ritter 2000, Scholz 1994, Scholz 1995, Thomas 1938)
- The first local bundle structures in physical context  
(Scholz 2001*a*, Scholz 2004)
- Topology and differential geometry: a rough first outlook  
(Berger 2000, Katz 1983, Chern 1971, Kolata 1979, James 1999, Dieudonné 1989)

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