Yield Uncertainty and Risk aversion in Duality models: a Mean-Variance approach

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Abstract

This paper aims at the exploration of the questions connected with the effects of uncertainty and risk in agriculture and, generally, in each economic sector where risk considerations are established. The aim of the first part is to give the key features for the estimation of production structures using dual approach and risk prevalence. In order to state risk non-neutrality into the implemented dual approach, Mean-variance utility function of revenue is introduced thereafter to be the objective function for the producer’s optimization process rather than revenue. Uncertainty was set as consequence of a weather variable variance and randomness of outputs’ quantities. The estimation is implemented on panel data of the Tunisian cereals sector from 1983 to 2005. The regional coefficients of risk aversion were highly significant and results from the Mean-Variance model shows several salient results relative to both weather and yield variability.

Key words: Agriculture, duality, Mean-variance utility, Risk aversion, Yield uncertainty.

JEL classification: C32, D81, Q11.
1 Introduction

In agricultural production, because of the time lag between input decision making and production realization, farm production decisions often depend on known input prices but uncertain output or price levels. The immediate implication of this uncertainty for economic agents is that many possible outcomes are usually associated with any one chosen action. Because not all possible consequences are equally desirable, decision-making under uncertainty is characterized by risk. Although uncertainty and risk always hold in economic activities, they constitute in agriculture an essential feature of the production environment and therefore deserve a detailed analysis.

Considerable research has been devoted to exploring questions connected with the effects of uncertainty and risk in agriculture, and these efforts have enhanced developments in the general economics literature.

Knowing that uncertainty in the agricultural activities holds by several features, it is useful to start by outlining the main sources of uncertainty and risk that are relevant from the point of view of the agricultural producer.

In agriculture the amount and quality of output that will result from a given bundle of inputs are typically not known with certainty, i.e., the production function is stochastic. This uncertainty is due to the fact that uncontrollable elements, such as weather, disease play a fundamental role in agricultural production. The effects of these uncontrollable factors are heightened by the fact that time itself plays a particularly important role in agricultural production, because long production lags are dictated by the biological processes that underlie the production of crops and the growth of animals. Although there are parallels in other production activities, it is fair to say that production uncertainty is a typical feature of agricultural production.

Price uncertainty is also a standard attribute of farming activities. Because of the biological production lags mentioned above, production decisions have to be made far in advance of realizing the final product, so that the market price for the output is typically not known at the time these decisions have to be made. Price uncertainty, of course, is all the more relevant because of the inherent volatility of agricultural markets. Such volatility may be due to demand fluctuations, which are particularly important when a sizable portion of output is destined for the export market. Production uncertainty as discussed earlier, however, also contributes to price uncertainty because price needs to adjust to clear the market. in this process some typical features
of agricultural markets (a large number of competitive producers, relatively homogeneous output, and inelastic demand) are responsible for generating considerable price volatility, even for moderate production shocks.

Additional sources of uncertainty are relevant to farming decisions when longer-term economic problems are considered. The randomness of new knowledge development affects production technologies in all sectors. What makes it perhaps more relevant to agriculture. It is due to the fact that technological innovations are the product of research and development efforts carried out elsewhere by firms supplying inputs to agriculture, such that competitive farmers are captive players in the process. Technical efficiency is an other feature that ought to bring a relevant deal of uncertainty to farmers who don’t know how they will be far from “best practices” actions under the available technology.

Policy uncertainty also plays an important role in agriculture. Again, economic policies have impacts on all sectors through their effects on such things as taxes, interest rates, exchange rates, regulation, provision of public goods, and so on. Yet, because agriculture in many countries is characterized by a complex system of government interventions, and because the need for changing these policy interventions in recent times has remained strong (witness the recent transformation of key features of the agricultural policy under the framework of the regional and/or multilateral commitments, or the emerging concerns about the environmental impacts of agricultural production), this source of uncertainty creates considerable risk for agricultural investments.

One of the most widely agreed upon results from the theory of the firm under price uncertainty is that risk affects the optimal output level. Normally, the risk-averse producer is expected to produce less than the risk-neutral producer, ceteris paribus, and the risk-averse producer will adjust output to changing risk conditions (e.g., decrease production as risk increases). Econometric studies of agricultural supply decisions have for a long time tried to accommodate these features of the theory of the firm. There are essentially two reasons for wanting to do so: first, to find out whether the theory is relevant, i.e., to “test” whether there is risk response in agricultural decisions; second, assuming that the theory is correct and risk aversion is important, accounting for risk response may improve the performance of econometric models for forecasting and/or policy evaluation, including welfare measurement related to risk bearing.

To pursue these two objectives, a prototypical model is to write supply
decisions conditional on (subjective) conditional expectation of price and its (subjective) conditional variance among other variables affecting decisions. Clearly, this formulation simplifies theory by choosing particular functional form and, more important, by postulating that mean and variance can adequately capture the risk facing producers.

Similar procedures have been very common in other studies, although often with the improvement of a weighted (as opposed to simple) average of squared deviations from the conditional (as opposed to unconditional) expectation of the price level (e.g., Lin, 1977; Traill, 1978; Hurt and Garcia, 1982; Sengupta and Sfeir, 1982; Brorsen, Chavas and Grant, 1987; Chavas and Holt, 1990, 1996). A more ambitious and coherent framework was proposed by Just (1974, 1976), whereby first and second moments of price are modeled to the same degree of flexibility by extending Nerlove’s (1958) notion of adaptive expectations to the variance of price. This procedure has been used in other studies, including Pope and Just (1991), Antonovitz and Green (1990), and Aradhyula and Holt (1990). More recently, advances have been made by modeling the time-varying variance within the autoregressive conditional heteroskedasticity (ARCH) framework of Engle (1982), as in Aradhyula and Holt (1989, 1990), Holt and Moschini (1992), and Holt (1993).

The empirical evidence suggests that risk variables are often significant in explaining agricultural production decisions. The early work by Just (1974), as well as some other studies, has suggested that the size of this supply response to risk may be quite large, but the quantitative dimension of this risk response is more difficult to assess because results are typically not reported in a standardized manner.

The models reviewed above introduce a risk variable as a single equation supply model. Representing risk in terms of a single variable (say, price variance) may be justified as an approximation to the more general EU model and will be an admissible procedure only under certain restrictive conditions (for example, normality and CARA). Whereas consideration of higher moments has been advocated by Antle and Goodger (1984) among others. The single equation nature of these supply models, on the other hand, can only be a partial representation of the more complete production and supply system that may represent the agricultural producer’s decision problem. Thus, generalizing risk response models to systems of equations may be desirable, and it has been pursued by Coyle (1992, 1999), Chavas and Holt (1990, 1996), and Saha, Shumway and Talpaz (1994), among others. Consideration of such complete supply systems is common in applied work under assumptions of
certainty or risk neutrality is due partly to the possibilities afforded by the
extension of flexible functional forms for dual representations of technology
such as profit and cost functions, which simplify the derivation of coherent
systems of output supply and input demand equations. Extension of the dual
approach under risk has been explored by Coyle (1992). His set-up relies on
a linear mean-variance objective function.

The system approach typically can accommodate integrability conditions
such as symmetry, homogeneity, and curvature (convexity in prices of the
profit function). Interest in these restrictions can arise for at least two rea-
sons. First, this set of testable restrictions may be used to validate the
theoretical framework. Second, if testing the theory is not an objective,
then maintaining these restrictions may be useful in improving the feasibil-
ity/efficiency of estimation, as well as improving the usefulness of empirical
results for policy and welfare analysis.

Pope (1980) pursued integrability conditions for EU maximizing produc-
ers and showed that the simple symmetry and reciprocity conditions that
hold under certainty need not hold under uncertainty. But, as in any opti-
mization problem, some symmetry conditions must exist, and for the case
of a producer who maximizes expected utility under price uncertainty, these
conditions were characterized by Pope (1980), Chavas and Pope (1985), and
Paris (1989). In general the relevant symmetry conditions will involve wealth
effects (and thus will depend on risk attitudes). Restrictions on preferences,
however, can reduce the symmetry and reciprocity conditions of the risk-
averse case to those of the certainty case.

2 A multi-stochastic-output model with land allocation

In deterministic production theory, the firm is assumed to maximize profit.
In von Neumann-Morgenstern (vNM henceforth) utility theory, it is assumed
to maximize its expected utility, where utility is a monotonically increasing,
continuously differentiable function of profit, income or any other measure of
wealth. Uncertainty may enter via stochastic production (input and output
quantities or yields) or stochastic prices.

If wealth, \( \pi \), is treated as a random variable with probability density
function, \( f(\pi) \), and the firm’s utility of profit is \( U(\pi) \), then the expected
utility of the firm is,

\[ E[U(\pi)] = \int_0^\infty U(\pi)f(\pi)d\pi \]

In the vNM utility theory, the objective of the firm is the maximization of \( E[U(\pi)] \).

The way to assess risk aversion considerations based on dual approach will addressed where dealing with risk features are taken into account by reshaping a standard dual model which leads to further theoretical specifications, comparing to a standard risk neutral dual model, since some properties of the dual approach doesn’t hold under such assumptions.

The duality model is developed here under the following assumptions: linear mean-variance risk preferences which implies constant absolute risk aversion (CARA), quadratic indirect utility function and price certainty. These assumptions regarding preferences and technology, albeit restrictive, have often been employed in empirical research in agriculture (e.g., Chavas and Pope 1982; Love and Buccola 1994; Coyle 1992, 1999 and more recently Sckokai and Moro, 2006). Moreover these assumptions imply that the firms objective function is almost linear in parameters, which simplifies exposition of the dual approach and, to some extent, simplifies empirical application.

As stated above, the duality model implies a general technology with multiple stochastic outputs. Risk preferences of the farmer are implemented assuming a mean-variance utility function where the certainty equivalent of the outcome is expressed in term of its two first moments: expectation and variance. this model implies implicitly some assumption regarding the distribution of the random component of the output and the functional form of the utility function such as a normal distribution for the former and a quadratic (direct) utility function for the latter. the certainly equivalent profit takes therefore the following form:

\[ EU(\pi) = \bar{\pi} - \frac{1}{2}\sigma^2 \pi \] (1)

where \( \bar{\pi} \) and \( \sigma^2 \) are the mean and variance of profit which is a random variable due to the stochastic aspect of the bundle of produced quantities. That is, randomness of profit is imputable entirely to the revenue component rather than cost component since both input prices and quantities are supposed deterministic.
Let consider a multi output multi input production process. The production process is characterized by a stochastic output level takes the following form:

\[ Y = G(X, l, \varepsilon) + u \]  

(2)

where \( Y \) is an \((n \text{ rows})\) output vector, \( G \) for a vector with terms \( g_i(X, l, \varepsilon) \) which are "well behaved" direct production functions, \( X \) is the \((m \text{ rows})\) input vector, \( l \) is the \((n \text{ rows})\) acreage allocation vector, \( \varepsilon \) a stochastic weather variable with mean \( \bar{\varepsilon} \) and variance \( \sigma^2_{\varepsilon} \). Indeed, the body literature in agricultural economics recognizes that the effect of weather conditions on biological processes governing agricultural products production is straightforward. Then, its introduction as an input or, since it is out of the control of farmers, as a state variable is effective to understand an agricultural production process. \( u \) is the vector of the random components of the outputs with mean 0 and covariance matrix \( \Omega \). Realizations of \( u \) occur due to the random events surrounding the production process other than weather condition embodied in \( \varepsilon \).

In order to identify the mean of the output level, each mean production function is expanded relative to \( \varepsilon \) using a second order Taylor series expansion around \( \bar{\varepsilon} \). Taking the mathematical expectation of the second order expansion, the expected output level is:

\[ \bar{Y} = F(X, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \]  

(3)

where \( F(X, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \) is a vector with terms \( f_i(X, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \).

It is assumed here that farmer who doesn’t observe \( \varepsilon \), makes his production planning conditional on the mean and variance of the weather variable. Output covariance matrix is then:

\[ \Omega_y = \Omega \]  

(4)

Recall that the profit is basically:

\[ \pi = \sum_{i=1}^{n} p_i y_i - \sum_{j=1}^{m} w_j x_j \]  

(5)

Note that profit is random. the mean and variance of profit are:
\[ \bar{\pi} = \sum_{i=1}^{n} p_i \bar{y}_i - \sum_{j=1}^{m} w_j x_j \]  
\[(6)\]

\[ \sigma^2_{\bar{\pi}} = P' \Omega P = \sum_{i=1}^{n} \sum_{k=1}^{n} p_i p_k \sigma_{ik} \]  
\[(7)\]

where \( P \) stands for the output price vector and \( \sigma_{ij} \) for covariance between the \( i \)th and the \( k \)th output. By inserting (3), (6) and (7) into (1), functional form of the certainly equivalent profit is obtained:

\[ EU(\pi) = \sum_{i=1}^{n} p_i f_i(X, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) - \sum_{j=1}^{m} w_j x_j - \frac{1}{2} \alpha P' \Omega P \]  
\[(8)\]

First order conditions for certainly equivalent profit maximization are given by:

\[ \frac{\partial EU(\pi)}{\partial x_j} = \sum_{i=1}^{n} p_i \frac{\partial f_i(X, l, \bar{\varepsilon}, \sigma^2_{\varepsilon})}{\partial x_j} - w_j = 0 \text{ for } j = 1..m \]  
\[(9)\]

(9) affords the necessary conditions to reach the optimal expected output and input levels, say \( \bar{y}_i(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \) and \( x_j(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \).

Optimal output and input are substituted into (8) and then the indirect utility function of profit can be retrieved:

\[ V(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}, \Omega) = \max_{x_j \geq 0} EU(\pi) = EU(\pi)^* = \sum_{i=1}^{n} p_i \bar{y}_i(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) - \sum_{j=1}^{m} w_j x_j(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) - \frac{1}{2} \alpha P' \Omega P \]  
\[(10)\]

where \( \pi^* \) is the maxim Making use of the Envelope Theorem (Takayama, p:137-39), partial derivatives of (10) with respect to the input and output prices give:

\[ \frac{\partial V(.)}{\partial p_i} = \bar{y}_i(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) + \sum_{k=1}^{n} p_k \frac{\partial \bar{y}_k(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon})}{\partial p_i} \]

\[ - \sum_{j=1}^{m} w_j \frac{\partial x_j(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon})}{\partial p_i} - \frac{1}{2} \alpha \frac{\partial (P' \Omega P)}{\partial p_i} \]

\[ = \bar{y}_i(P, W, l, \bar{\varepsilon}, \sigma^2_{\varepsilon}) \]
\[ + \sum_{j=1}^{m} \left[ \left( \sum_{k=1}^{n} p_k \frac{\partial \bar{y}_k(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2)}{\partial x_j} - w_j \right) \frac{\partial x_j}{\partial p_i} \right] \]

\[ - \alpha \sum_{k=1}^{n} p_k \sigma_{ik} \]

\[
\frac{\partial V(.)}{\partial w_j} = -x_j(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2) + \sum_{k=1}^{n} p_k \frac{\partial \bar{y}_k(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2)}{\partial w_j} \\
- \sum_{j=1}^{m} w_j \frac{\partial x_j(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2)}{\partial w_j} \\
= -x_j(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2) + \sum_{j=1}^{m} \left[ \left( \sum_{k=1}^{n} p_k \frac{\partial \bar{y}_k(P, W, l, \bar{\varepsilon}, \sigma_{\bar{\varepsilon}}^2)}{\partial x_j} - w_j \right) \frac{\partial x_j}{\partial w_j} \right]
\]

Equations (13) and (14) are an important finding for the duality under risk. Indeed they imply that functional forms for output supply and input demand equations can be retrieved based on the formulation stated above. Even if Hotelling’s Lemma does not hold when the dual model is reshaped to account for risk non neutrality, optimal output supply and input demand functions can be retrieved by the virtue of the Envelope Theorem. Note that equation (14) relating input demands to derivatives of the dual with respect to input prices is identical to what can be retrieved by the standard Hotelling’s lemma under risk neutrality. This occurs because uncertainty here is embodied in the output level; All the input side variables and output prices being deterministic. hence, there is no effect of risk aversion behavior on the demand structure. Besides, (13) is slightly different from a supply function obtained assuming a standard dual approach. the second term in the right hand side of (13) is the effect of risk non neutrality on output supply.
Equations (13) provides a specification of expected output supply equation in terms of the dual: it relates output supply $\bar{y}$ to the dual’s derivative of $V(.)$ relative to $p$.

The indirect utility function $V(P,W,l,\bar{z},\sigma_z^2,\Omega)$ should be consistent both with vNM and neoclassical theory. The following Proposition establishes the properties of the system obtained:

**Proposition 1** Assuming the existence of an indirect utility function $V(.)$. For the system of output supply and input demand derived from $V(.)$ to be consistent with vNM and neoclassical theories, the following properties must be verified.

- **a)** \( \frac{\partial V(.)}{\partial p_i} + \alpha \sum_{k=1}^{n} p_k \sigma_{ik} \geq 0 \) for \( i = 1..n \). That is, $V(.)$ is non-decreasing in $P$.
- **b)** \( -\frac{\partial V(.)}{\partial w_j} \geq 0 \) for \( j = 1..m \). That is, $V(.)$ is non-increasing in $W$.
- **c)** $V(\lambda P, \lambda W, l, \bar{z}, \sigma_z^2, \Omega) = \lambda V(P, W, l, \bar{z}, \sigma_z^2, \Omega)$. i.e., $V(.)$ is homogeneous of the degree 1 in output and input prices.
- **d)** The Hessian matrix of $V(.)$ or more generally $V(.) + P'\Omega P$ with respect to $P$ and $-W$ is positive semi-definite.

While the two first properties of Proposition 1 are straightforward and well established by analogy with the standard duality theory, properties b and c need further explications.

Property c indicates that the dual $V(.)$ is linear homogeneous in $(P, W)$ that is, decisions $(x, \bar{y})$ are homogeneous of degree zero in $(P, W)$. Hence, knowing that first derivative engenders a decrease in the degree of homogeneity, Homogeneity of the degree 1 of the dual utility function produces homogeneous of degree zero supply and demand functions. Positive semi-definiteness of $V(.) + P'\Omega P$ implies that the terms on the diagonal of the Hessian should not be negative, i.e.

\[
\frac{\partial^2 V}{\partial p_i^2} = \frac{\partial \bar{y}}{\partial p_i} - \alpha \sigma_{ii} \geq 0 \quad \text{for } i = 1..n \\
\frac{\partial^2 V}{\partial w_j^2} = -\frac{\partial x_j}{\partial w_j} \geq 0 \quad \text{for } j = 1..m
\]

Proof for positive semi-definiteness of $V(.)$ under risk aversion

\[\text{Proof positive semi-definiteness of } V(.) \text{ under risk aversion}^1\]
The standard dual model under risk non neutrality was presented above. Since dual models are basically consistent with multi output processes, acreage allocation rules ought to be derived. In the recent literature many authors have sought to model the question of area allocation and crop specific input levels and output quantities within a dual framework (Coyle, 1992, Lansink and Peerlings, 1996 and Lansink, 1999). The issue is tackled in order to derive optimal acreage allocation equations as a part of the decision making rule.

Based on the indirect (dual) utility function (10), additional first order conditions for the maximization of $V(.)$ with respect of acreage allocations for each crop $l_i$ for $i = 1..n$ would imply \(^2\):\[
\frac{\partial V}{\partial l_i} = \frac{\partial V}{\partial l_j} = 0
\] (17)

(17) implies that optimal land allocation between crops is attended when marginal profit relative to land allocation is null, that is there is no gain in changing land allocation. the allocation is done subject to the constraint that the total land area is fixed in the short run.

After conceptual framework for a dual model under linear mean variance assumption being established, the aim of the next section is to provide an empirical assessment using panel data of the Tunisian cereals sector.

### 3 Empirical assessment

As stated above, duality model can, under some assumptions, provide useful model that can be used in empirical implementation. The use of the model developed in section 2 is twofold: first to assess the conformity of the model can be found in Coyle, 1999 who has extensively discussed the issue.

\(^2\)This specification implies that the shadow price of land is null. Note that (17) can easily be reshaped by the deviation of shadow prices of acreage allocation from zero. By the way, both parametrizations are similar in empirical emplimentation since (17) provides a linear acreage allocation equations and the shadow price will be embodied in the intercept of these equations.
when confronted to data and therefore to provide an empirical proof of its consistency. Second, to explore the behaviors of Tunisian cereals producers under risk non neutrality. Indeed, apart from Dhif and Ben Jemaa (2004), there are a priori no econometric models dealing with duality under risk aversion for Tunisian agriculture. Tunisia is geographically situated in an arid and semi-arid area leading crop yields highly volatile. Cereal crops are among the most sensitive crops to climate and production technology such as intra-annual rainfall repartition, temperature and quantities of chemical inputs used (fertilizers and pesticides), etc.

Due to the lack of data on input use, except for land allocation to each crop, the interest in the empirical issue is turned to the implementation of the revenue instead of the profit as a measure of wealth. The adoption of the revenue as a measure of wealth is consistent with the mean-variance model stated in section 2 since input prices are not uncertain in Tunisia. Hence, the consideration of a revenue function preserves risk non neutrality against output levels’ uncertainty.

The model assumes three outputs: durum wheat, barely and tender wheat, which are the widely cropped cereals in Tunisia\(^3\). Land is introduced into the model as acreage allowed to each crop in order to account for land allocation under fixed total acreage (at least in the short term). The three output levels are assumed uncertain.

Data used in the empirical illustration are a panel data of Output quantities, land allocation for each crop and monthly measures for rainfall precipitations of the 24 “gouvernorats” in Tunisia from 1981 to 2005.

A problem arises when one aims at the estimation of production and rain expectations, variances and covariances. The most logical manner is obviously to compute empirical first and second moments of these variables. By doing so, it will be argued that producers’ anticipations about variables levels is based on their historical realizations. There are many ways to compute these parameters such the use of weighted (increasing) averages and variances of the three past realizations as proxies for means and variances (and covariances) respectively (Chavas and Holt, Coyle, 1992 and 1999, Dhif and Ben Jemaa 2004 among others)\(^4\). In order to preserve the maximum of the

\(^3\)Around 90 percent of total cereals land in Tunisia is devoted to these three crops. Durum wheat and barely monopolize both about 80 percent of total broad acre land.

\(^4\)Breman (1982) has compared, within a standard mean vari-
time span of the data set at disposal, expected values are approximated as a
two years (one and two lagged) moving average. Variances and covariances
are approximated as a two years (one and two lagged) moving variance.

Although production data is available at the “gouvernorat” level, data on
monthly rainfall level is available only for eighteen weather stations within
Tunisia. As a consequence, Some “gouvernorats”, that belong to the same
bioclimatic region, share the same data on rain precipitation levels. It is
widely accepted that the distribution of rain within a year has a great impact
on cereals’ yields. That was the reason leading to use a weighted average of
monthly data on precipitation instead of an equal weighting average. In order
to estimate these weights, cereals yields are regressed on monthly levels of
precipitation (from September to Mai). Estimates are normalized by their
sum. Then, the obtained coefficients, that add up to unity, are used to
compute a weighted average of the weather variable for each panel unit.5

Mean and variance of weather at time $t$ were calculated as a weighted
past realizations;

\[
\overline{\epsilon}_t = 0.5(\epsilon_{t-1} + \epsilon_{t-2}) \tag{18}
\]

\[
\sigma^2_{\epsilon_t} = 0.5 \sum_{i=1}^{2} (\epsilon_{t-i} - \overline{\epsilon}_t)^2 \tag{19}
\]

Mean expression (18) fits rational expectation when believes at time $t$ are
an average of past realizations.

Expected output levels and variance are approximated using the same
protocol as in (18) and (19). not that the protocol of the expectations of
ance model, seven different measures of price variability as prox-
ies for risk. These measures included three and four periods
moving averages and standard deviations and measures of the
magnitude of the difference between expected and actual prices.
His conclusion was that there is no deep difference between all
the used protocols.

5By the way, it was found that precipitations in fabruary have
the greatest marginal effect on cereals’ yield instead of march as
it is commonly confirmed. This finding is due to the fact that
research on seed amelioration has produced during the three
last decades varieties that grows up relatively early in order to
escape from weather variability and rainfall irregularity by which
is characterized the spring of Tunisia.
output levels needs further explications. It is worth to recall that expectations of output levels are set conditional, among others, on observed acreage allocations. Thus, informations about acreage allocations are used as a supplementary support to better parametrize expected value of production. As a matter of fact, farmers can not allow land for a crop when they do expect null production and, ipso facto, they do not expect null production for a crop when they do allow really acreage for it. Based on this argument, expected output productions are reshaped accordingly to be:

$$\bar{y}^k_t = \begin{cases} 
0.5(\varepsilon_{t-1} + \varepsilon_{t-2}) & \text{if } l^k_t > 0 \\
\hat{r}^k_t \tilde{l}^k_t & \text{if } 0.5(\varepsilon_{t-1} + \varepsilon_{t-2}) \equiv 0 \text{ and } l^k_t > 0 \\
0 & \text{if } l^k_t \equiv 0 
\end{cases} \text{ for } k = 1, 2, 3 \quad (20)$$

Where $\hat{r}^i_j$ stands for the estimated yield from the regression of crop yields on monthly precipitations levels$^6$.

Covariance of outputs levels are approximated using the following expression:

$$\sigma_{ks} = 0.5 \sum_{i=1}^{2} \sum_{j=1}^{2} (y_{k_{t-i}}^i - \bar{y}^i_t)(y_{s_{t-j}}^j - \bar{y}^j_t) \quad (21)$$

Prominent among the functional forms commonly used to describe a profit or a revenue function is the normalized quadratic (NQ) specification (Villezca-Becerra and Shumway, 1994; Guyomard et al., 1996) which allows linear equations for the examined dependent variables (except for the price used as numéraire). Partial derivatives with respect to output prices and area allocations provide the flowing system of simultaneous equations based on equations (13) and (17) and assuming a normalized quadratic dual utility function:

$$\bar{y}^k_{it} = a_{ks} + \sum_{h=1}^{2} a_{kh} p_{ith} + \sum_{h=1}^{2} b_{kh} l_{ith} \beta_{1k} \varepsilon_{it} + \beta_{2k} \sigma_{it}^2 + \beta_{3k} p^*_{it} + \sum_{j=1}^{2} \alpha_{kj} D_j Q_{itk} + \eta_{itk} \quad (22)$$

$$l^k_{it} = c_{ks} + \sum_{h=1}^{2} d_{kh} p_{ith} + e_k l_{ith} + \delta_{1k} \varepsilon_{it} + \delta_{2k} \sigma_{it}^2 + \delta_{3k} p^*_{it} + \nu_{itk} \quad (23)$$

$^6$Data on yields are obviously censored at zero. That’s why panel data tobit regression was used to estimate the model of monthly rain effect on crops’ yields.
Where the $D_j$’s are two dummy variables indicating whether the $i$’th “gouvernorat” is situated in the north or in the south. Thus the model allows for the estimation of different coefficients of absolute risk aversion for the northern and for the central-southern “gouvernorats”. This specification allows the comparison of risk aversion between the two regions. The $Q_{itk}$’s are estimated linear prices-variance-covariance products $\sum_{k=1}^{3} p_{itk}\sigma_{itk}$ which are the risk variables. The $\eta_{itk}$’s and $\upsilon_{itk}$ are random errors.

In order to insure the theoretical consistency of the model to be estimated, adding up restrictions is obtained by dropping output supply and land allocation equations of tender wheat. The price of tender wheat $p_{it}^*$ is used as a numéraire for output prices and risk variables. The numéraire is introduced in the model as a covariate in order to test for the homogeneity of equations (22) and (23).

In order to handle the heterogeneity of the units in the panel data, a group fixed effects are introduced to each equation of the model (22)-(23). The choice of the fixed effect instead of a random effect specification is due to the nature of panel data. Indeed the panel data is not obtained by a random sampling process; the sample contains all the “gouvernorats” in Tunisia and then discrepancies between “gouvernorats” is deterministic and permanent.

The $a_{ks}$’s and the $c_{ks}$’s are groups’ fixed effects for expected outputs and land allocations equations respectively. The “gouvernorats” are divided into four geographic regions: North east (Tunis, Ariana, Ben arous, Manouba, Zaouzouan, Nabeul and Bizerte), North-west (Beja, Jendouba, Kef and Seliana), center (Kasserine, Gafsa, Sidi Bouzid, Kairouan, Sfax, Sousse, Monastir and Mahdia) and south (Tozeur, Kebili, Mednine, Gabes and Tataouine).

The symmetry and reciprocity properties consistent with the standard neoclassical theory can be imposed with the following cross equations restrictions: $a_{kh} = a_{hk}$, $b_{kh} = b_{hk}$, $d_{kh} = d_{hk}$, $e_i = e_j$, $\alpha_{1j} = \alpha_{2j}$.

The last constraint implies that the coefficient of absolute risk aversion should not vary across output since it is unique for a producer and it does not depend on produced commodities.

### 4 Estimations and Main results

The data on “gouvernorats” of Tunisia concerning production levels, land allocation, output prices and rainfall covering the period 1983-2005 is com-
piled from various issues of the Agricultural Statistics year book from the Department of Planning and Statistics of the Tunisian Ministry of agriculture. The presence of zero production and zero land allocation in the data, which implies corner solution problem, requires the use of censored regression techniques such as Amemiya-Tobin model based on probability regimes and Maximum (simulated) likelihood estimation (Demeke and Coxhead, 2005; Pitt and Millimet, 2003, Dong and Kaiser, 2003 among others) or a two step estimation procedure (Shonkwiler and Yen, 1999). However, observations presenting censored outcome does represent only 7 percent of the total number of observations. Then, the data set was truncated and confined only to those units who simultaneously recorded non zero production and allocation for both durum wheat and barely.

In order to account for the endogenous nature of acreage allocation variables and the impact of cross equations linear constraints, cross equations correlation of the error terms is assumed for the $i$'th unit in the current period. This implies that the variance-covariance matrix of the errors terms $\mathbb{E} \left[ (\eta_k, \upsilon_k)' (\eta_k, \upsilon_k) \right]$, where $(\eta_k, \upsilon_k)$ is a row vector of errors term of the totality of the observations, is a bloc-diagonal matrix and the bloc is a four dimensional variance-covariance matrix of the models error terms $\eta_k$ and $\upsilon_k$ for $k = 1, 2$.

A Three stage Least Square was used to estimate parameters of the model (22)-(23) and all the cross equations restrictions are taken into account. Estimates are reported in Appendix 1-table A2. About 63 percent of the estimates are significant at least at the 10 percent significance level. As anticipated under risk aversion and crops’ yield uncertainty, the coefficients $\beta_{2k}$ (for $i = 1, 2$) of weather variance in the crops’ output equations are negative but only the effect on durum wheat supply is significant. The significant negative sign for the $\beta_{1k}$ implies that expected crops’ output supply are decreasing in weather variance, as expected under risk aversion and yield uncertainty. The significance of weather variance suggests that this model deals both with direct and indirect effect of random environment on outputs supply. Indeed the variance of the weather variable can be seen as a risk variable and then the introduction of mean and variance of the weather variable within the model under mean variance fashion can be considered as nested.

Although it seems that no gain ought to be obtained by handling corner solutions in the data set, this issue will be tackled in a forthcoming research.
specifications of risk assessment. For land allocation equations, the coefficient associated to weather variance in durum wheat acreage allocation $\delta_{21}$ is significantly negative which implies the same interpretation as for the same output supply function. However, $\delta_{22}$ was found to be positive but not significant. This statement confirms that weather variance has no impact either on output supply or on acreage allocation for barely. While weather variability is land reducing for durum wheat, this variability increases significantly the acreage allocated to barely which is well known as resisting cereal to climatic variations. The increase of acreage allocated to barely in droughty years is a well known strategy to cope with risk in such “bad” years. The coefficient associated to expected value of the weather variable (the $\beta_{1k}$’s and $\delta_{1k}$’s) are significantly positive except for land allocation equation of barely which is significantly negative. This statement comes to corroborate the specificity of barely as a resistant crop widely cropped in arid areas.

On the other hand, estimates of the coefficients of risk aversion $\alpha_{ij}$ were found to be highly significant and with the right positive sign. The coefficient of absolute risk aversion for the north was found significantly smaller than one for the south.

As mentioned above, the numéraire $p^*$ is introduced in the equations as a covariate. the $\beta_{3k}$’s were found non significant which imply that output demand equations are homogeneous of the degree zero as suggested by the theory. The homogeneity restrictions under the standard risk-neutral model and under CARA are rejected for land allocation equation since the numéraire’s coefficients $\delta_{3k}$ are significant.

Price elasticities presented in Appendix 1-Table 2 are partial price elasticities: they represent the intensity effect of prices, acreages, expected weather variable and its variance change. All of the own and cross price elasticities of output quantities and land allocation have the correct sign theoretically, that is positive definiteness of the Hessian implies that all own output price elasticities are positive.

The inclusion of the area allocation helps to capture the full complexity of the supply response (Appendix 1-Table 2). For instance, an increase in the output prices for durum wheat and barely results in an increase in there land shares.

Three nested variants of the model was estimated in order to test its robustness. Model b is a model in which risk neutrality is assumed that is risk variables were dropped from supply functions. In model c, dependent variables are explained only by group fixed effects. Model d is a model under
risk aversion in which no distinction is made between degree of risk aversion for the north and the south, ie, $\alpha_1 = \alpha_2$. As it can be seen in Appendix 1-Table 3, The original model dominates significantly specifications b, c and d based on an LR test. The AIC and BIC criteriums come to corroborate the superiority of the model (Appendix 1 - table 4).

4.1 Concluding remarks

In sum, within the framework of the model considered here, uncertainty about weather conditions influences production decisions. These impacts on short-run production decisions are small as indicated by the elasticity estimates in Appendix 1- table 2. However, these impacts are statistically significant and may improve estimation of models’ deterministic components.

Estimate of Absolute Risk Aversion coefficients $\alpha_j$’s were significant for both regions which is the main strong point of the model comparing to previous works that failed to prove significant Absolute Risk Aversion coefficient (Coyle, 1999;McQuinn, 2000). Note that the coefficient of absolute risk aversion for the south was about 5 times greater than the north.

In the framework of this paper, it was question to modelize risk non-neutrality into a dual approach. Mean-variance utility function of profit was introduced in section to be the objective function for the producer’s optimization process rather than profit. Uncertainty was set as consequence of a weather variable variance and randomness of outputs’ quantities.

The main innovation in this model was the implementation of a multi-stochastic-output function which implies the consideration of covariances between random output quantities.

Weather variable variance was found to have a negative impact on expected supplied quantity for durum wheat but its impact was found to be non significant on both supply and land allocation of barely. Estimate of the Absolute Risk Aversion coefficients were highly significative and had the right theoretical sign which is the main strong point of the model comparing to previous works that failed to prove significant Absolute Risk Aversion coefficient. Although a set of good results, the model isn’t free of several shortcomings. Using aggregated data rather than farm-level data is not the best information for risk assessment and ought to lead to number of criticisms.
References


Appendix

Table 1-a: 3 SLS estimates

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Std. Err.</th>
<th>t ratio</th>
<th>p. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{20}$</td>
<td>-1.559</td>
<td>1.267</td>
<td>-1.230</td>
</tr>
<tr>
<td>$a_{1ne}$</td>
<td>-1.291</td>
<td>0.583</td>
<td>-2.210</td>
</tr>
<tr>
<td>$a_{1nw}$</td>
<td>4.031</td>
<td>2.429</td>
<td>1.660</td>
</tr>
<tr>
<td>$a_{1cr}$</td>
<td>0.494</td>
<td>0.795</td>
<td>0.620</td>
</tr>
<tr>
<td>$a_{1st}$</td>
<td>0.889</td>
<td>0.501</td>
<td>1.780</td>
</tr>
<tr>
<td>$a_{11}$</td>
<td>1.075</td>
<td>0.154</td>
<td>6.960</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-0.463</td>
<td>0.182</td>
<td>-2.540</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>0.645</td>
<td>0.406</td>
<td>1.590</td>
</tr>
<tr>
<td>$b_{12}$</td>
<td>-0.266</td>
<td>0.309</td>
<td>-0.860</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.507</td>
<td>0.118</td>
<td>4.570</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
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<td>0.025</td>
<td>-3.210</td>
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<td>$\beta_{31}$</td>
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<td>0.374</td>
<td>-1.030</td>
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<td>0.001</td>
<td>4.290</td>
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<tr>
<td>$\alpha_2$</td>
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<td>0.002</td>
<td>3.600</td>
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Table 1-b: 3 SLS estimates

<table>
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<th>p. value</th>
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<td>$a_{20}$</td>
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<td>$a_{2cr}$</td>
<td>1.805</td>
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<tr>
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<td>-0.970</td>
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<tr>
<td>$a_{21}$</td>
<td>-0.463</td>
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<td>$a_{22}$</td>
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<td>0.170</td>
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<td>$\beta_{32}$</td>
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<td>0.090</td>
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<td>$\alpha_1$</td>
<td>0.003</td>
<td>0.001</td>
<td>4.290</td>
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<tr>
<td>$\alpha_2$</td>
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<td>0.002</td>
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### Table 1-c: 3 SLS estimates

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<tr>
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<tr>
<td>$d_{11}$</td>
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<tr>
<td>$e_{1}$</td>
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<td>0.000</td>
</tr>
<tr>
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### Table 1-d: 3 SLS estimates

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<tr>
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<td>3.860</td>
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<td>$c_{2ne}$</td>
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<td>-3.940</td>
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<td>$c_{2nw}$</td>
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<td>0.000</td>
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<td>$\delta_{22}$</td>
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<td>0.037</td>
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Table 2: Price, land and weather variable Elasticities*

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<tr>
<th></th>
<th>Y D. wheat</th>
<th>Y barely</th>
<th>Land D. wheat</th>
<th>Land barely</th>
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</thead>
<tbody>
<tr>
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<td>-0.61</td>
<td>0.17</td>
<td>-0.27</td>
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<tr>
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<tr>
<td>Y barely</td>
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<td>0.06</td>
<td>-0.28</td>
<td>0.18</td>
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<tr>
<td></td>
<td>0.06</td>
<td>0.35</td>
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<td>0.07</td>
</tr>
<tr>
<td>Land D. wheat</td>
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<td>-0.59</td>
<td>-</td>
<td>1.53</td>
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<tr>
<td></td>
<td>0.37</td>
<td>0.68</td>
<td>-</td>
<td>0.02</td>
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<tr>
<td>Land barely</td>
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<td>0.65</td>
<td>-</td>
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<tr>
<td></td>
<td>0.44</td>
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<td>0.01</td>
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<tr>
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<td>0.48</td>
<td>0.83</td>
<td>0.22</td>
<td>-0.33</td>
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<td></td>
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<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>( \sigma^2_{\bar{\varepsilon}} )</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.02</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.07</td>
<td>0.01</td>
<td>0.02</td>
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*All Elasticities are evaluated at the sample mean.
Standard errors are under coefficients.

Table 3: Nested specifications tests
Ho: No supplementary gain in a with respect to:

<table>
<thead>
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<th>Specification</th>
<th>LR</th>
<th>df</th>
<th>Decision on Ho</th>
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<tbody>
<tr>
<td>b</td>
<td>482.17</td>
<td>2</td>
<td>Rejected</td>
</tr>
<tr>
<td>c</td>
<td>3888.2</td>
<td>26</td>
<td>Rejected</td>
</tr>
<tr>
<td>d</td>
<td>1177.88</td>
<td>1</td>
<td>Rejected</td>
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</table>

Table 4: Log-Likelihood, AIC and BIC

<table>
<thead>
<tr>
<th>Model</th>
<th>N(^2) of obs</th>
<th>Log L</th>
<th>df</th>
<th>AIC</th>
<th>BIC</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>464</td>
<td>-1902.016</td>
<td>46</td>
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<tr>
<td>c</td>
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<td>7815.031</td>
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