Pricing Warrants Models: An Empirical Study of the Indonesian Market

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Wajih ABBASSI²

Abstract

The main issue during periods of financial crisis is to restore the investor’s confidence and attract them back to financial markets. Thus, the warrants have been a great help to relaunch many south East Asian financial markets just after the 1997 crisis, by encouraging investors to finance the restructuring of troubled companies. However, the few empirical studies which were interested in this product, was limited to the developed markets and in particular the American one. In this paper, we will try to emphasize the warrant’s specificities compared to the option ones. Then, we will seek to release the ideal model ready to provide the best pricing for this product. The comparison will relate to 3 models: the Black and Sholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and finally the modified Square Root Constant Elasticity of Variance (SRCEV) adjusted by the dilution effect. The empirical study, which is spread out over 3 years (2001–2003), will relate to an emergent market: the Jakarta Stock Exchange (JSX).

Key words: Black and Scholes, Constant elasticity variance, Dilution, Indonesian market, Moneyness, Warrant valuation.

JEL Classification: G12, G13

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Introduction

Warrants are financial instruments issued by financial institutions and are generally negotiated on the options markets. A warrant holder has the right to buy or to sell a specific underlying asset once arrived the warrant expiry at a fixed price called the strike price. At first sight, one can confuse warrant with conventional option. Both share the same principles of underlying asset, strike price and expiry date. Moreover, several papers used to valuate warrant price as same as option’s one using black and Scholes formula. However, there are two major differences between warrants and conventional options. First, the warrant lifetime is quite large than the option’s one. A warrant can have a time to expiration going up to seven years whereas an option commonly has a time to expiration not exceeding few months. Second, options are issued by individuals so an eventual exercise will only lead to the underlying asset transfer from one operator to another one. Warrants are issued by firms. Consequently, their exercise will cause the issue of additional shares and thus a firm equity dilution.

In spite of warrants success, a few papers were involved on their valuation. Moreover, most of these papers were focused on the developed markets. For this paper, we are interested on empirical validation of three valuation models for warrants issued on an emergent market. We select the Indonesian market for 2 reasons. First, because of the south East Asian financial market dynamic character this doesn’t cease attracting all kinds of investors and speculators. Also, the Indonesian warrant market recorded the highest growth rate, in term of stock exchange capitalization, compared to other Indonesian financial markets, particularly after the 1997 crisis. At those circumstances, warrants were issued to encourage reinvesting in Indonesian stocks.

We will try to compare the performances of three warrant valuation models using the data observed on the Jakarta Stock Exchange for the years 2001, 2002 and 2003. The models tested are the Black and Scholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and the modified Square Root Constant Elasticity Variance model (modified SRCEV) inspired from Cox (1975)\(^3\) Constant Elasticity Variance Model (CEV).

This paper will be structured as follows. The first section introduces the BS and DABS models. The second introduces the CEV model and the necessary modifications, in order to make the Cox Model more suitable for warrants valuation. The third section introduces the statistical summaries of the data used through our empirical study. The fourth section compares the BS and DABS valuation performances. The fifth deals about the underlying assets volatility behaviour. The sixth section compares the modified SRCEV and DABS valuation performances. Finally, we conclude by a brief summary of the main results and some suggestion for future research.

I- Valuating warrants with BS and DABS models

1. **DABS model: Galai and Schneller approach (1978)**\(^4\)

Like a conventional European call, the warrant is the right to buy, at the strike price, an underlying asset at an expiration date. However, warrants are issued by companies so that their exercise will lead to an issue of additional shares and, therefore, a dilution of the issuing company equities. We try to emphasize the relation between the issuing firm value and the warrant price.

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\(^3\) Warrants have appeared for the first time on the Jakarta Stock Exchange dated July 13, 1995 and were issued by the bank TIARA ASIA.

\(^4\) J. Cox (1975). “Notes on option pricing: Constant Elasticity Variance Diffusions”. Working Paper, Stanford University. Through this paper, Cox has established a new model for pricing options assuming that volatility, is no longer constant, but moves inversely to the underlying asset price.

Let consider a firm (A) with 100% of equities and (n) shares. Suppose that this firm will be liquidated at date (T) with a stochastic future value $V_T$. Each share value, at that date, will be $S_T = \frac{V_T}{n}$.

Now let consider a second firm (B), identical to the first, that issues (m) warrants at t=0, with strike price K and expiration date T. Each exercised warrant allows its holder to possess a new issuing firm share. So, the conversion ratio, noted $\lambda$, is equal to 1. If warrants are exercised at expiration date, the firm B value will be higher than the firm (A) one.

$$V_T^* = V_T + m \cdot K$$ (1)

The share value of the firm (B) at time T, noted $S_T^*$, will be equal to

$$S_T^* = \frac{V_T + m \cdot K}{n + m}$$ (2)

The warrant will be exercised at expiration date, only if the underlying stock price exceeds the warrant strike price.

$$S_T^* > K \Rightarrow \frac{V_T + m \cdot K}{n + m} > K$$ (3)

In other hand, $S_T = \frac{V_T}{n}$, so

$$S_T^* = \frac{n \cdot S_T + m \cdot K}{n + m} > K$$ (4)

Now, let try to infer the warrant price using the BS European call pricing formula. Consider a European call with the firm (A) share as an underlying asset. At expiration date, the call value will be equal to $\text{Max}(0, S_T - K)$. At the same date, the warrant value will be equal to $\text{Max}(0, S_T^* - K)$.

$$S_T^* - K = \frac{n \cdot S_T + m \cdot K}{n + m} - K = \frac{n}{n + m}(S_T - K)$$ (5)

If $S_T > K$, both the call and the warrant will be exercised, so

$$W_T = \frac{n}{n + m} \cdot C_T$$ (6)

If $S_T < K$, the call and the warrant will be abandoned and will have a null value.

We conclude that the warrant and call prices are perfectly correlated. In an efficient market, and with the no arbitrage assumption, 2 financial assets offering 2 perfectly correlated outputs will have their prices correlated with the same coefficient at any time $t \in [0; T]$. So

$$W_t = \frac{n}{n + m} \cdot C_t$$ (7)

As we substitute $C_t$ by the BS formula, the warrant value can be written as

$$W_t = \frac{n}{n + m} \cdot [S_t \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)]$$ (8)
Consider a conversion ratio $\lambda$, the DABS warrant pricing formula as presented Galai and Schneller is

$$W_t = \frac{n}{\lambda} \left[ \left( S_t + \frac{m}{n} \cdot w \right) \cdot N(d_1) - K \cdot e^{-r\tau} \cdot N(d_2) \right]$$  \hspace{1cm} (9)$$

$W_t$ : the warrant price at time (t)

$S_t$ : the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two bonds whose maturities straddle the warrant expiration date.

$\tau$: the warrant time to expiration

$$d_1 = \ln \left( \frac{S_t + \frac{m}{n} \cdot w}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau$$

$$d_2 = d_1 - \sigma \cdot \sqrt{\tau}$$

$m$: number of issued warrants

$n$: number of underlying shares

$w$: the warrant price at the issue date

$\lambda$: Conversion ratio, i.e. for each warrant exercised, $\lambda$ new underlying shares are created.

2. **BS model: Bensoussan, Crouhy and Galai approach (1995)**

This approach assumes that the underlying asset price reflects the dilution effect. The current shareholders anticipate the effect of an eventual dilution at the issue announcement. At that date, there would be a jump in the underlying asset price. That’s why, Bensoussan, Crouhy and Galai consider that there is no need to make modifications to the BS model when it comes to evaluating warrants because the dilution effect is directly included into the underlying asset price. The warrant theoretical price will be

$$W_t = S_t \cdot N(d_1) - K \cdot e^{-r\tau} \cdot N(d_2)$$  \hspace{1cm} (10)$$

$W_t$ : the warrant price at time (t)

$S_t$ : the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two T-bills whose maturities straddle the warrant expiration date.

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6 Indeed, whenever a warrant holder decides to exercise, the issuing company will be required to create $\lambda$ new shares for him.

\( \tau \): the warrant time to expiration

\[
\ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau
\]

\[
d_1 = \frac{\ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau}{\sigma \cdot \sqrt{\tau}}
\]

\[
d_2 = d_1 - \sigma \cdot \sqrt{\tau}
\]

\( N(.) \): the cumulative normal distribution

**II- The modified Square Root Constant Elasticity Variance model**

An extremely practical consequence of the underlying asset price lognormality assumption is that the historical variance can be used to predict the future volatility and thus to evaluate options. Unfortunately, empirical had proven that neither the historical nor the implied volatility can give a future variance estimate which is unique and constant over time. Indeed, there are few chances that the historical variance calculated over the five last years is equal to the historical variance calculated over the last six months. There is considerable evidence in the literature indicating that stock returns are heteroscedastic, with a probability distribution showing a negative skewness. Black (1976)\(^8\) writes “I have believed for long time that stock returns are related to volatility changes. When stocks go up, volatilities seem to go down; and when stocks go down, volatilities seem to go up”.

Cox (1975) proposed a call valuation model which assumed that the volatility is related, by a negative and constant relation, to the asset price. The diffusion process characterizing the Cox model takes the form

\[
\frac{dS}{S} = \mu \cdot dt + \delta \cdot S^{\theta} \cdot dz
\]

\( \mu \): the expected asset return rate

\( \delta \cdot S^{\theta} \): the instantaneous standard deviation of the return rate asset, with \( \delta \) and \( \theta \) being constants

\( dz \): a Wiener standard process

The elasticity variance \( h_s \) with respect to the stock price is

\[
h_s = \left( \frac{\partial \sigma^2}{\partial S} \right) \left( \frac{S}{\sigma^2} \right) = \frac{\partial \left( \delta^2 \cdot S^{\theta-2} \right)}{\partial S} \frac{S}{\delta^2 \cdot S^{\theta-2}} = \theta - 2
\]

It is easily seen that the model CEV will be equivalent to the BS model when \( \theta = 2 \) and that the volatility is a decreasing function of \( S \), as reported by Black (1975), if \( \theta < 2 \). Under the differential equation (11) and the set as assumptions in the BS framework\(^9\), Cox (1975) derived the equilibrium price formula of European call option\(^{10}\) for \( \theta < 2 \). We can clearly see that the elasticity \( h_s \) is constant

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\(^9\) Although the variance is no longer stationary, according to the equation (11), it’s still a deterministic (constant) function of the asset price. The Cox model is a one state variable model, which is once again the asset price. Under these circumstances, it’s possible to establish a perfectly covered position using the call and its underlying asset. When applying the no arbitrage assumption, it becomes possible to obtain the call pricing formula. Moreover, this formula can be applied the risk preferences. Indeed, in a neutral risk economy, individuals require no risk premium. All assets have an expected return rate equal to the risk free interest rate.

\(^{10}\) Cox option pricing formula is developed in appendix A.
and negative. That’s why the Cox model is often called the Constant Elasticity Variance (CEV) model. The CEV European call price details are presented in the appendix.

Several empirical studies, Emanuel and Mac Beth (1982), Bates (1995) and Jones (2003), had shown that the CEV model corrects a systematic bias of the BS model which tends to overprice the in-the-money calls and underprice the out-of-the-money ones. However, when the volatility is inversely related to the asset price, high (low) level asset prices must have low (high) volatilities, so the in-the-money (out-of-the money) calls CEV prices will be lower (higher) than the BS ones.

As the warrant life term is generally higher than one year, we should use a valuation model that allows variance to vary through time. For that reason, using CEV model to pricing warrants should be more appropriate than the use of the BS model. To calculate the CEV warrant price, we introduced 2 main modifications to the general formula set up by Cox. First, we fix $\theta = 1$. This choice will considerably simplify the Cox formula. More, several studies, Beckers (1980)$^{11}$, Lauterbach and Schultz (1990)$^{12}$, Hauser and Lauterbach (1997)$^{13}$, provide encouraging estimates of warrants and options prices when keeping $\theta = 1$. The CEV model will be transformed on what’s called “Square Root Constant Elasticity Variance” (SRCEV) model. (See the formula details in appendix). The second modification made to the CEV model is that we integrated the dilution effect as we did for the DABS model.

The modified SRCEV model is

$$W_t = \frac{1}{n} \left[ \left( S_t + \frac{m}{n} \cdot w \right) \cdot N[q(4)] - K \cdot e^{\sigma^2 \cdot t} \cdot N[q(0)] \right]$$ (13)

$q(4)$ and $q(0)$ are developed in appendix B.

IV- Database

The database used for this study consists on:

- Warrants prices: the end-of-the day prices for the warrants negotiated on the Jakarta Stock Exchange (JSX) during the period going from 01/02/2001 until the 12/31/2003. We count 85 call warrants. Theses prices were directly obtained from the JSX communication and public relationship department.

- Information bulletins: also obtained from the JSX communication and public relationship department. These bulletins indicate, for each warrant, the issue date, the exercise price, the expiry date. The conversion ratio is equal to 1 for all the warrants negotiated on the JSX during this period.

- Underlying shares prices: the end-of-the day shares prices. The total number of the observed prices is 29978. However, neither the database, obtained from the JSX communication and

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$^{12}$ B. Lauterbach and P. Schultz. “Pricing Warrants: An Empirical Study of the Black Scholes Model and Its Alternatives”. Journal of Finance (1990). This paper, considered as the first one that dealt, in an exhaustive manner, the issue of warrant pricing using the DABS and the SRCEV models.

$^{13}$ S. Hauser and B. Lauterbach (1997). “The Relative Performance of Five Alternative Warrant Pricing Models”. Financial Analysts Journal. This paper was interested in comparing the performance of five pricing warrants models traded at the American market for the period 1971 to 1980. The models used were the BS model, the DABS model, the Longstaff model for extended warrants, the SRCEV model and the Ritchken free theta model. The latter gave the lowest estimation error (3.6%) ahead of the SRCEV model (3.67%) ranked in 2nd position.
public relationship department, nor that downloaded from the web site www.jsx.co.id provides information about dividend distribution.

- Riskless Interest Rates: the interest rates for governmental obligations issued during 2001, 2002 and 2003. The information were downloaded from Indonesian Central Bank web site: www.bi.go.id.

Several filters were used in order to reach a sample that can be used for a reliable analysis. We eliminate warrants with incomplete information bulletin. Then, we eliminate warrants inappropriate for the pricing models used through our paper, i.e. the warrants with a “set up” exercise price following a stock split operation. We also apply a liquidity filter. We retain warrants with a liquidity level higher than 20%. The warrants prices that do not match the no arbitrage assumption are excluded from the sample. Finally, we keep the warrants that have, at least, 100 useable observations. The summarized statistics for the final sample are presented at table 1. The distribution of the observation according to the criteria of moneyness\(^{14}\) and time to expiration are presented at table 2. It’s noted that the final sample is dominated by deep out and out-of-the-money warrants, which account for 83.61% of the total observations. Regarding the time to expiration, 73.06% of the observations show a time to expiration beyond a year versus 23.7% with a time to expiration less than a year.

V- “Dilution Adjusted Black & Scholes” vs. “Black & Scholes”

We calculate the estimation error as the absolute deviation of the market warrant price from the theoretical price, reported to the warrant price. This measure illustrates the exactness with which each model fits the observed prices. The average estimation error (AEE), for the whole observations, can be presented as follows

\[
AEE = \frac{1}{n} \sum_{j=1}^{n} \left| \frac{W_{j_{\text{model}}} - W_{j_{\text{market}}}}{W_{j_{\text{market}}}} \right|
\]

Where \((n)\) is the number of observed warrant prices across all warrants and days of the sample.

The AEE, calculated through 5460 observations, are 14.45% and 12.76% respectively for BS and DABS models. Applying the z-test, the minimum significant difference test, we find that the 2 AEE are significantly different from each other at level \(\alpha=1\%\).

Now, let calculate the AEE by warrant. Table (3) provides the results. It also provides the factors that may explain the performance of each of the 2 models.

Except for the warrant TMPO-W, the DABS model provides better estimates than BS model. This superiority can be explained by the fact that DABS model succeeded to correctly treat the dilution effect by making an explicit adjustment of the BS model, via the introduction of the dilution factor \(\frac{n}{n + m}\). An implicit adjustment for the dilution effect through the underlying asset price, as assumed by Bensoussan, Crouhy and Galai (1995), doesn’t seem very suitable for the Indonesian market. We clearly see that the highest difference, between the estimation errors of each model, was

\(^{14}\) Moneyness is calculated as \(\frac{S_t - K \cdot e^{-rt}}{K \cdot e^{-rt}}\) where \(S_t\) is the underlying asset price, \(K\) the warrant exercise price, \(r\) the riskless interest rate and \(\tau\) the time to expiration.
recorded for the warrant that has the highest dilution factor (warrant PLAS-W with a dilution factor of 74%). On the other side, the smallest difference was recorded for the warrant with the smallest dilution factor (warrant ITTG-W with a dilution factor of 9%).

We also classify the observation into 9 subsamples using the warrant moneyness and time to expiration criteria. We made same calculus and same comparison between the 2 models for each of these categories. Results are summarized in table 4.

We clearly see that the DABS average estimation error get closer to the BS one as we move from out-of-the money warrants subsamples to in-the-money ones. Moreover, when applying the z-test, we find that the DABS estimation errors are significantly different from the BS ones (significance at level α=5%) only for deep out-of-the money subsamples. Such a result can be explained by the fact that these warrants are precisely these who have the highest dilution factor (see table 1).

Another finding is that, both DABS and BS models perform the worst for out and deep out-of-the money warrants, exactly as do the classic BS option valuation formula. This is due to the fact that these 2 models still use the underlying asset price lognormality assumption with a constant volatility. Such an assumption is hardly suitable for a long life term asset as the warrant is.

VI- the volatility Behaviour

As shown in table 4, the AEE varies, dramatically, as we move cross subsamples. This may be the result of strong relationship between the local volatility rate and the asset price. To confirm this assumption, we can regress the implied volatility on the moneyness factor. The existence of a significant negative relation between the asset price and its volatility will, necessarily, reinforce the position of the modified SRCEV model versus the BS and DABS ones. To have a more clear idea on this issue, we apply the following regression:

\[
ISD_t = \alpha_0 + \alpha_1 \cdot \left( \frac{S_t - K \cdot e^{-rT}}{K \cdot e^{-rT}} \right)_t + \epsilon_t \quad (15)
\]

Where \( ISD_t \) is the warrant implied standard deviation calculated at date \( t \). \( \left( \frac{S_t - K \cdot e^{-rT}}{K \cdot e^{-rT}} \right)_t \) is the moneyness factor.

First of all, we do the regression for the whole panel data, across all warrants. We apply three different panel regression models: Fixed Effects Model, Weighted Fixed Effects Model and Random Effects Model. The regression results are summarized at table 5.
### Table 1 - DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Presence in the sample</th>
<th>Obs.(^1)</th>
<th>Dilution</th>
<th>Expiration</th>
<th>Strike Price</th>
<th>Mean « Moneyness »</th>
<th>Mean ISD(^2)</th>
<th>Mean time to expiration</th>
<th>Life Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>10/08/01 – 12/30/03</td>
<td>385</td>
<td>28.57%</td>
<td>07/19/2004</td>
<td>300</td>
<td>-0.363</td>
<td>0.4759</td>
<td>1.487</td>
<td>3 years</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>01/18/02 – 12/30/03</td>
<td>317</td>
<td>33.33%</td>
<td>01/18/2005</td>
<td>250</td>
<td>-0.261</td>
<td>0.9276</td>
<td>1.759</td>
<td>3 years</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>06/12/01 – 12/30/03</td>
<td>397</td>
<td>12.5%</td>
<td>06/07/2004</td>
<td>250</td>
<td>-0.02</td>
<td>0.3668</td>
<td>1.38</td>
<td>3 years</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>04/16/02 – 12/31/03</td>
<td>380</td>
<td>44.44%</td>
<td>11/22/2004</td>
<td>125</td>
<td>-0.894</td>
<td>1.3365</td>
<td>1.83</td>
<td>3 years</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>08/12/02 – 12/30/03</td>
<td>319</td>
<td>20%</td>
<td>08/11/2005</td>
<td>275</td>
<td>-0.455</td>
<td>0.5059</td>
<td>2.314</td>
<td>3 years</td>
</tr>
<tr>
<td>IDS-R-W</td>
<td>03/22/01 – 12/30/03</td>
<td>602</td>
<td>16.67%</td>
<td>03/21/2004</td>
<td>650</td>
<td>0.169</td>
<td>0.9304</td>
<td>1.457</td>
<td>3 years</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>11/26/01 – 12/31/03</td>
<td>490</td>
<td>9.09%</td>
<td>11/25/2004</td>
<td>150</td>
<td>-0.324</td>
<td>0.4829</td>
<td>1.964</td>
<td>3 years</td>
</tr>
<tr>
<td>KARK-W</td>
<td>07/20/01 – 12/30/03</td>
<td>575</td>
<td>48.45%</td>
<td>07/19/2004</td>
<td>125</td>
<td>-0.704</td>
<td>0.819</td>
<td>1.799</td>
<td>3 years</td>
</tr>
<tr>
<td>KREN-W</td>
<td>06/28/02 – 12/30/03</td>
<td>347</td>
<td>16.67%</td>
<td>06/28/2005</td>
<td>265</td>
<td>-0.577</td>
<td>0.5507</td>
<td>2.258</td>
<td>3 years</td>
</tr>
<tr>
<td>META-W</td>
<td>07/18/01 – 07/10/02</td>
<td>239</td>
<td>50%</td>
<td>07/17/2002</td>
<td>200</td>
<td>-0.481</td>
<td>1.139</td>
<td>0.503</td>
<td>1 year</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>03/16/01 – 12/30/03</td>
<td>585</td>
<td>74.07%</td>
<td>03/15/2004</td>
<td>200</td>
<td>-0.538</td>
<td>0.6317</td>
<td>1.473</td>
<td>3 years</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>01/08/02 – 12/31/03</td>
<td>451</td>
<td>44.44%</td>
<td>01/07/2004</td>
<td>300</td>
<td>-0.394</td>
<td>0.7342</td>
<td>1.06</td>
<td>2 years</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>01/02/02 – 12/30/03</td>
<td>466</td>
<td>20%</td>
<td>06/21/2004</td>
<td>175</td>
<td>-0.754</td>
<td>1.1501</td>
<td>1.492</td>
<td>2.5 years</td>
</tr>
</tbody>
</table>

\(^1\) The number of observations of each warrant in the sample.  
\(^2\) The implied standard deviation.
**Table 2 - Distribution of the Observations according to the moneyness and the time to expiration criteria**

The distribution of the 5460 observations will be based on 2 criteria: the moneyness and the time to expiration. Figures in parentheses are the percentages of observations of each subsample compared to the whole sample. Deep in-the-money warrants are those with a moneyness factor $>0.5$. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5;-0.05]$. Deep out-of-the money warrants have a moneyness factor $<-0.5$.

<table>
<thead>
<tr>
<th></th>
<th>≤ 1 year</th>
<th>&gt; 1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deep In the money</strong></td>
<td>38 (0.70%)</td>
<td>103 (1.89%)</td>
</tr>
<tr>
<td><strong>In the money</strong></td>
<td>155 (2.83%)</td>
<td>206 (3.77%)</td>
</tr>
<tr>
<td><strong>At the money</strong></td>
<td>202 (3.70%)</td>
<td>1508 (27.62%)</td>
</tr>
<tr>
<td><strong>Out of the money</strong></td>
<td>899 (16.47%)</td>
<td>1941 (35.55%)</td>
</tr>
</tbody>
</table>

**Table 3 - Comparison of Estimation Errors by Warrant**

A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors.

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Observations</th>
<th>Mean Moneyness</th>
<th>Dilution</th>
<th>Error BS</th>
<th>Error DABS</th>
<th>Performance BS – DABS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>347</td>
<td>-0.363</td>
<td>28.57%</td>
<td>14.73%</td>
<td>14.28%</td>
<td>0.46%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>299</td>
<td>-0.261</td>
<td>33.33%</td>
<td>3.56%</td>
<td>3.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>360</td>
<td>-0.02</td>
<td>12.50%</td>
<td>7.15%</td>
<td>5.27%</td>
<td>1.88%</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>379</td>
<td>-0.894</td>
<td>44.44%</td>
<td>21.40%</td>
<td>18.34%</td>
<td>3.06%</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>316</td>
<td>-0.455</td>
<td>20%</td>
<td>13%</td>
<td>12.30%</td>
<td>0.70%</td>
</tr>
<tr>
<td>IDSR-W</td>
<td>601</td>
<td>0.169</td>
<td>16.67%</td>
<td>5.07%</td>
<td>4.91%</td>
<td>0.16%</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>487</td>
<td>-0.324</td>
<td>9.09%</td>
<td>11.93%</td>
<td>11.82%</td>
<td>0.11%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>570</td>
<td>-0.704</td>
<td>48.45%</td>
<td>20.80%</td>
<td>18.82%</td>
<td>1.98%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>344</td>
<td>-0.577</td>
<td>16.67%</td>
<td>15.47%</td>
<td>14.94%</td>
<td>0.53%</td>
</tr>
<tr>
<td>META-W</td>
<td>236</td>
<td>-0.481</td>
<td>50%</td>
<td>24.28%</td>
<td>22.88%</td>
<td>1.40%</td>
</tr>
<tr>
<td></td>
<td>BS</td>
<td>DABs</td>
<td>Difference</td>
<td>Probability*17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>-------</td>
<td>-------</td>
<td>------------</td>
<td>----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep In 1 year</td>
<td>4.44%</td>
<td>4.87%</td>
<td>-0.43%</td>
<td>0.2562</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In 1 year</td>
<td>5.31%</td>
<td>5.26%</td>
<td>0.05%</td>
<td>0.469</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In - 1 year</td>
<td>25.58%</td>
<td>35.76%</td>
<td>-10.18%</td>
<td>0.2789</td>
<td></td>
<td></td>
</tr>
<tr>
<td>At - 1 year</td>
<td>6.86%</td>
<td>5.71%</td>
<td>1.15%</td>
<td>0.2726</td>
<td></td>
<td></td>
</tr>
<tr>
<td>At + 1 year</td>
<td>10.07%</td>
<td>9.92%</td>
<td>0.15%</td>
<td>0.4859</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>11.72%</td>
<td>10.71%</td>
<td>1.01%</td>
<td>0.1312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out - 1 year</td>
<td>14.55%</td>
<td>12.99%</td>
<td>1.56%</td>
<td>0.293</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>16.2%</td>
<td>14.13%</td>
<td>2.07%</td>
<td>0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>22.2%</td>
<td>18.31%</td>
<td>3.89%</td>
<td>0.0107</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis probabilities for the z-test.
Table 5 - Comparative Table of Regression Models

Comparative table aimed to identify the estimation results given by each regression model. The criteria used in this comparison are: the calculated (t) student, the regression linearity coefficient and the sum of residual squares. We compare the Fixed Effects Model (FEM), the weighted FEM and the Random Effects Model (REM).

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>FEM</th>
<th>weighted FEM</th>
<th>REM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.455</td>
<td>-1.331</td>
<td>-1.480</td>
</tr>
<tr>
<td>$t(\alpha_1)$</td>
<td>-64.708</td>
<td>-99.205</td>
<td>-83.555</td>
</tr>
<tr>
<td>$R^2$</td>
<td>61.69%</td>
<td>78.21%</td>
<td>70.28%</td>
</tr>
<tr>
<td>$R^2_{\text{adjusted}}$</td>
<td>61.6%</td>
<td>78.15%</td>
<td>70.28%</td>
</tr>
<tr>
<td>SCR</td>
<td>506.673</td>
<td>382.446</td>
<td>393.009</td>
</tr>
</tbody>
</table>

All regression models show a strong inverse relationship between implied volatility and moneyness. The three $\alpha_1$ estimations are significantly negative at level $\alpha=1\%$. The best regression model seems to be the Weighted Fixed Effects Model providing the most significant $\alpha_1$ estimation with the highest “t” student, the best linearity and adjusted linearity coefficients and the lowest sum of squared residuals. Such a result suggests that the modified SRCEV model would be more appropriate to fit warrants prices than DABS and BS models.

According to stochastic volatility models, Wiggins (1987)<sup>18</sup>, Hull and White (1987)<sup>19</sup>, Heston (1993)<sup>20</sup>, the volatility varies through time without being linked to the underlying asset price by a deterministic relation. In other terms, these models allow arbitrary correlation between volatility and asset price. Such volatility behaviour, as shown by Heston (1993), produces a probability distribution of the asset returns with high kurtosis. Thus, the relation between the implied volatility and the underlying asset price can vary as the warrant is in or out-of-the money.

To verify whether the volatility is purely stochastic or not, we repeat the regression, using the weighted Fixed Effect Model, only for in-the-money warrants. The in-the-money observations represent 12% of the whole panel (665 observations among a total of 5460 warrants prices). The regression results are summarized in table 6.

---


Table 6 - Regression results for in-the-money observations

<table>
<thead>
<tr>
<th>Warrant</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>0.1442</td>
<td></td>
</tr>
<tr>
<td>ANTA-W</td>
<td>0.3029</td>
<td></td>
</tr>
<tr>
<td>BCAP-W</td>
<td>0.2636</td>
<td>-0.1742</td>
</tr>
<tr>
<td>IDSRY-W</td>
<td>0.6177</td>
<td></td>
</tr>
<tr>
<td>ITTY-W</td>
<td>0.2629</td>
<td></td>
</tr>
<tr>
<td>KARK-W</td>
<td>0.1735</td>
<td></td>
</tr>
<tr>
<td>META-W</td>
<td>0.1753</td>
<td></td>
</tr>
<tr>
<td>PLAS-W</td>
<td>0.1245</td>
<td></td>
</tr>
<tr>
<td>TMPO-W</td>
<td>0.1483</td>
<td></td>
</tr>
</tbody>
</table>

$R^2 = 93.81\%$  \hspace{1cm}  $R^2_{\text{adjusted}} = 93.72\%$

Although the inverse relation is more pronounced for out-of-the-money warrants, the regression has shown that such relation is maintained for in-the-money ones with an $\alpha_1$ estimation significantly negative at level $\alpha = 1\%$. Then, we can deduce that the volatility remains inversely related to the underlying asset price, as assumed by the SRCEV model, and not purely stochastic.

VII- Modified “SRCEV” vs. “DABS”

The AEE, calculated through the 5460 observations, are 8.95% and 12.76% respectively for modified SRCEV and DABS models. Such a result was predictable as the modified SRCEV is adapted to the 2 warrant specificities, i.e. the dilution effect and the warrant long life term, by incorporating the dilution factor in the original pricing formula and using a variable volatility. The $z$-test shows that the 2 AEE are significantly different from each others at level $\alpha = 1\%$.

Table 7 compares the AEE calculated for each warrant using the 2 models.

Table 7 - Comparison of Estimation Errors by Warrant

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Average Moneyness</th>
<th>Error DABS</th>
<th>Error SRCEV</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>-0.363</td>
<td>14.275%</td>
<td>8.20%</td>
<td>6.075%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>-0.261</td>
<td>3.33%</td>
<td>0.47%</td>
<td>2.86%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>-0.02</td>
<td>5.27%</td>
<td>5.27%</td>
<td>0.01%</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>-0.894</td>
<td>18.34%</td>
<td>14.13%</td>
<td>4.21%</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>-0.455</td>
<td>12.3%</td>
<td>8.85%</td>
<td>3.45%</td>
</tr>
<tr>
<td>IDS-R-W</td>
<td>0.169</td>
<td>4.91%</td>
<td>3.49%</td>
<td>1.42%</td>
</tr>
<tr>
<td>ITT-G-W</td>
<td>-0.324</td>
<td>11.82%</td>
<td>11.29%</td>
<td>0.53%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>-0.704</td>
<td>18.82%</td>
<td>9.81%</td>
<td>9.01%</td>
</tr>
<tr>
<td>KRENS-W</td>
<td>-0.577</td>
<td>14.94%</td>
<td>10.17%</td>
<td>4.77%</td>
</tr>
<tr>
<td>META-W</td>
<td>-0.481</td>
<td>22.876%</td>
<td>15.31%</td>
<td>7.566%</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>-0.538</td>
<td>12.72%</td>
<td>12.14%</td>
<td>0.58%</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>-0.394</td>
<td>13.89%</td>
<td>9.72%</td>
<td>4.17%</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>-0.754</td>
<td>14.42%</td>
<td>8.59%</td>
<td>5.83%</td>
</tr>
</tbody>
</table>

Now let calculate the AEE across subsamples made upon the moneyness and the time to expiration criteria. Results are summarized at table 8.

We find that the modified SRCEV estimation errors are significantly different, at level $\alpha = 5\%$, from the DABS errors, for the out-of-the-money subsamples. On the other hand, we can see that the 2 models estimation errors are not significantly different from each others, for at and in-the-money warrants. The 2 models provide an almost similar performance for these categories of warrants.

Such a result can be explained by the modified SRCEV assumption for fixed $\theta = 1$ across all the moneyness categories and for all warrants. This assumption may be suitable for out-of-the money warrants, where the modified SRCEV model had performed its best performance, but it seems that it doesn’t work so good for at and in-the-money subsamples. For these categories of warrants, there is still an inverse relation between volatility and asset price (as shown in the previous section) but the correlation between these 2 variables should be more moderate than for out-of-the money warrants. Investors tend to be very frightened face any downtrend in the market and to demonstrate a very moderate optimism when the movement becomes bullish. Such behaviour is quite remarkable response to periods of financial crises. The volatility decreases monotonically as the asset price goes upward but the rate of decrease should be less than the rate of volatility increase when asset price goes downward. Thus $\theta$ must vary as we move from out-of-the money to in-the-money subsamples.
Table 8 - Estimation Errors classified by Moneyness and Time to expiration

A summary table which aims to compare the performances provided, respectively, by the DABS model and the SRCEV model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor>0.5. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [-0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5;0.05]$. Deep out-of-the money warrants have a moneyness factor<0.5.

<table>
<thead>
<tr>
<th></th>
<th>DABS</th>
<th>SRCEV</th>
<th>Difference</th>
<th>Probability$^{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.87 %</td>
<td>3.98 %</td>
<td>0.89 %</td>
<td>0.318</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.26 %</td>
<td>4.36 %</td>
<td>0.9 %</td>
<td>0.411</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>35.76 %</td>
<td>14.76 %</td>
<td>21 %</td>
<td>0.116</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>9.92 %</td>
<td>9.01 %</td>
<td>0.91 %</td>
<td>0.420</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>5.71 %</td>
<td>5.54 %</td>
<td>0.17 %</td>
<td>0.1574</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>10.71 %</td>
<td>8.26 %</td>
<td>2.45 %</td>
<td>0.0384</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>12.99 %</td>
<td>11.03 %</td>
<td>1.96 %</td>
<td>0.0005</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>18.31 %</td>
<td>10.89 %</td>
<td>7.42 %</td>
<td>0</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>14.13 %</td>
<td>9.72 %</td>
<td>4.41 %</td>
<td>0</td>
</tr>
</tbody>
</table>

Conclusion

As expected, the modified SRCEV model led to the lowest AEE for the whole panel, for each warrant and for the different moneyness subsamples. This result can be explained by the fact that modified SRCEV is the only one of the 3 models used in our empirical study, which tries to exploit the 2 major warrant specificities.

First, the modified SRCEV is explicitly adjusted for the dilution effect which should allow offsetting the negative impact that dilution could have on the quality of estimation.

Also, unlike BS and DABS models, the modified SRCEV is more adapted to the warrant long life term, proposing a diffusion process with variable underlying asset price volatility. This assumption seems to be more consistent than that of constant volatility used by BS and DABS models.

However, the modified SRCEV model may be subject to some criticism. We can refer, in particular, its use of a constant elasticity variance across all the underlying assets. This elasticity can even vary for the same underlying asset depending if the spot price is going upward or downward.

To remedy this deficiency, one can calculate the implied elasticity using the observed warrants prices and then calibrate the modified SRCEV model so it can be adapted for the underlying asset specificities.

$^{21}$ The null hypothesis probabilities for the z-test.
Bibliography


Appendix A - Cox Option Pricing Formula

Cox option pricing formula under the assumption of a constant elasticity variance is defined in the following manner:

\[
C_v = S \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1) \cdot G(\lambda \cdot (K \cdot e^{-rT})^n, n+1 - \frac{1}{\phi}) - K \cdot e^{-rT} \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1 - \frac{1}{\phi}) \cdot G(\lambda \cdot (K \cdot e^{-rT})^n, n+1)
\]

\(C_v\) is a European call price with a strike price \(K\) and a time to expiration \(\tau = (T - t)\);
\(\phi = 2 \theta - 2\);
\(\lambda = 2 \frac{\delta}{\delta^2} \cdot \phi \cdot e^{(r \phi t - 1)}\);
\(\Gamma(n) = \int_0^\alpha e^{-v} \cdot v^{n-1} \cdot dv\) : the gamma function;
\(g(x, n) = e^{-x} \cdot x^{n-1} \cdot \frac{1}{\Gamma(n)}\) : the density gamma function;
\(G(a, n) = \int_0^\alpha g(x, n) \cdot dx\) : the complementary gamma distribution function.

Appendix B - SRCEV option pricing formula as proposed by Beckers (1980)

For \(v=0\) or \(v=4\),
\[
(v) = \frac{1 + h(h-1) \cdot p - \frac{1}{2} \cdot h(h-1)(2-h)(1-3h) \cdot p^2 - \frac{z}{(v+y)}^h}{[2h^2 \cdot p \cdot [1-(1-h)(1-3h)p]]^{0.5}},
\]
\[
h(v) = 1 - \frac{2(v+y)(v+3y)}{3(v+2y)^2}, \quad p(v) = \frac{(v+2y)}{(v+y)^2}, \quad y = \frac{4r \cdot (S + \frac{M}{N} \cdot W)}{\sigma^2(1-e^{-rT})} \quad \text{et} \quad z = -\frac{K \cdot y}{S + \frac{M}{N} \cdot W}.
\]
Pricing Warrants Models: An Empirical Study of the Indonesian Market

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Abstract

The main issue during periods of financial crisis is to restore the investor’s confidence and attract them back to financial markets. Thus, the warrants have been a great help to relaunch many south East Asian financial markets just after the 1997 crisis, by encouraging investors to finance the restructuring of troubled companies.

However, the few empirical studies which were interested in this product, was limited to the developed markets and in particular the American one. In this paper, we will try to emphasize the warrant’s specificities compared to the option ones. Then, we will seek to release the ideal model ready to provide the best pricing for this product. The comparison will relate to 3 models: the Black and Sholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and finally the modified Square Root Constant Elasticity of Variance (SRCEV) adjusted by the dilution effect. The empirical study, which is spread out over 3 years (2001–2003), will relate to an emergent market: the Jakarta Stock Exchange (JSX).

Key words: Black and Scholes, Constant elasticity variance, Dilution, Indonesian market, Moneyness, Warrant valuation.

JEL Classification: G12, G13

Theme: Financial crisis and asset pricing

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Introduction

Warrants are financial instruments issued by financial institutions and are generally negotiated on the options markets. A warrant holder has the right to buy or to sell a specific underlying asset once reached the warrant expiry at a fixed price called the strike price. At first sight, one can confuse warrant with conventional option. Both share the same principles of underlying asset, strike price and expiry date. Moreover, several papers used to valuate warrant price as same as option’s one using black and Scholes formula. However, there are two major differences between warrants and conventional options. First, the warrant lifetime is quite large than the option’s one. A warrant can have a time to expiration going up to seven years whereas an option commonly has a time to expiration not exceeding few months. Second, options are issued by individuals so an eventual exercise will only lead to the underlying asset transfer from one operator to another one. Warrants are issued by firms. Consequently, their exercise will cause the issue of additional shares and thus a firm equity dilution.

In spite of warrants success, a few papers were involved on their valuation. Moreover, most of these papers were focused on the developed markets. For this paper, we are interested on empirical validation of three valuation models for warrants issued on an emergent market. We select the Indonesian market\(^1\) for 2 reasons. First, because of the south East Asian financial market dynamic character this doesn’t cease attracting all kinds of investors and speculators. Also, the Indonesian warrant market recorded the highest growth rate, in term of stock exchange capitalization, compared to other Indonesian financial markets, particularly after the 1997 crisis. At those circumstances, warrants were issued to encourage reinvesting in Indonesian stocks.

We will try to compare the performances of three warrant valuation models using the data observed on the Jakarta Stock Exchange for the years 2001, 2002 and 2003. The models tested are the Black and Scholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and the modified Square Root Constant Elasticity Variance model (modified SRCEV) inspired from Cox (1975)\(^5\) Constant Elasticity Variance Model (CEV).

This paper will be structured as follows. The first section introduces the BS and DABS models. The second introduces the CEV model and the necessary modifications, in order to make the Cox Model more suitable for warrants valuation. The third section introduces the statistical summaries of the data used through our empirical study. The fourth section compares the BS and DABS valuation performances. The fifth deals about the underlying assets volatility behaviour. The sixth section compares the modified SRCEV and DABS valuation performances. Finally, we conclude by a brief summary of the main results and some suggestion for future research.

I- Valuating warrants with BS and DABS models

1. **DABS model: Galai and Schneller approach (1978)\(^5\)**

Like a conventional European call, the warrant is the right to buy, at the strike price, an underlying asset at an expiration date. However, warrants are issued by companies so that their exercise will lead to an issue of additional shares and, therefore, a dilution of the issuing company equities. We try to emphasize the relation between the issuing firm value and the warrant price.

---

1. Warrants have appeared for the first time on the Jakarta Stock Exchange dated July 13, 1995 and were issued by the bank TIARA ASIA.
2. J. Cox (1975). “Notes on option pricing: Constant Elasticity Variance Diffusions”. Working Paper, Stanford University. Through this paper, Cox has established a new model for pricing options assuming that volatility, is no longer constant, but moves inversely to the underlying asset price.
Let consider a firm (A) with 100% of equities and (n) shares. Suppose that this firm will be liquidated at date (T) with a stochastic future value $V_T$. Each share value, at that date, will be $S_T = \frac{V_T}{n}$.

Now let consider a second firm (B), identical to the first, that issues (m) warrants at t=0, with strike price K and expiration date T. Each exercised warrant allows its holder to possess a new issuing firm share. So, the conversion ratio, noted $\lambda$, is equal to 1. If warrants are exercised at expiration date, the firm B value will be higher than the firm (A) one.

$$V'_T = V_T + m \cdot K \quad (1)$$

The share value of the firm (B) at time T, noted $S'_T$, will be equal to

$$S'_T = \frac{V_T + m \cdot K}{n + m} \quad (2)$$

The warrant will be exercised at expiration date, only if the underlying stock price exceeds the warrant strike price.

$$S'_T > K \quad \Rightarrow \quad \frac{V_T + m \cdot K}{n + m} > K \quad (3)$$

In other hand, $S_T = \frac{V_T}{n}$, so

$$S'_T = \frac{n \cdot S_T + m \cdot K}{n + m} > K \quad (4)$$

Now, let try to infer the warrant price using the BS European call pricing formula. Consider a European call with the firm (A) share as an underlying asset. At expiration date, the call value will be equal to Max (0 ; $S_T$ -K). At the same date, the warrant value will be equal to Max (0 ; $S'_T$ -K).

$$S'_T - K = \frac{n \cdot S_T + m \cdot K}{n + m} - K = \frac{n}{n + m}(S_T - K) \quad (5)$$

If $S_T > K$, both the call and the warrant will be exercised, so

$$W_T = \frac{n}{n + m} \cdot C_T \quad (6)$$

If $S_T < K$, the call and the warrant will be abandoned and will have a null value.

We conclude that the warrant and call prices are perfectly correlated. In an efficient market, and with the no arbitrage assumption, 2 financial assets offering 2 perfectly correlated outputs will have their prices correlated with the same coefficient at any time $t \in [0;T]$. So

$$W_t = \frac{n}{n + m} \cdot C_t \quad (7)$$

As we substitute $C_t$ by the BS formula, the warrant value can be written as

$$W_t = \frac{n}{n + m} \left[ S_t \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \right] \quad (8)$$
Consider a conversion ratio $\lambda$, the DABS warrant pricing formula as presented Galai and Schneller is

$$W_t = \frac{n}{\lambda} + m \cdot \left[ S_t + \frac{m}{n} \cdot w \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) \right] $$

$W_t$: the warrant price at time (t)

$S_t$: the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two bonds whose maturities straddle the warrant expiration date.

$\tau$: the warrant time to expiration

$$d_1 = \frac{\ln \left( \frac{S_t + \frac{m}{n} \cdot w}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau}{\sigma \cdot \sqrt{\tau}}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{\tau}$$

$m$: number of issued warrants

$n$: number of underlying shares

$w$: the warrant price at the issue date

$\lambda$: Conversion ratio, i.e. for each warrant exercised, $\lambda$ new underlying shares are created.

2. **BS model: Bensoussan, Crouhy and Galai approach (1995)**

This approach assumes that the underlying asset price reflects the dilution effect. The current shareholders anticipate the effect of an eventual dilution at the issue announcement. At that date, there would be a jump in the underlying asset price. That’s why; Bensoussan, Crouhy and Galai consider that there is no need to make modifications to the BS model when it comes to evaluating warrants because the dilution effect is directly included into the underlying asset price. The warrant theoretical price will be

$$W_t = S_t \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) $$

$W_t$: the warrant price at time (t)

$S_t$: the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two T-bills whose maturities straddle the warrant expiration date.

---

4 Indeed, whenever a warrant holder decides to exercise, the issuing company will be required to create $\lambda$ new shares for him.

\( \tau \): the warrant time to expiration

\[
\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau
\]

\[
d_1 = \frac{\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau}{\sigma \cdot \sqrt{\tau}}
\]

\[
d_2 = d_1 - \sigma \cdot \sqrt{\tau}
\]

\( N(.) \): the cumulative normal distribution

II- The modified Square Root Constant Elasticity Variance model

An extremely practical consequence of the underlying asset price lognormality assumption is that the historical variance can be used to predict the future volatility and thus to evaluate options. Unfortunately, empirical had proven that neither the historical nor the implied volatility can give a future variance estimate which is unique and constant over time. Indeed, there are few chances that the historical variance calculated over the five last years is equal to the historical variance calculated over the last six months. There is considerable evidence in the literature indicating that stock returns are heteroscedastic, with a probability distribution showing a negative skewness. Black (1976)\(^8\) writes “I have believed for long time that stock returns are related to volatility changes. When stocks go up, volatilities seem to go down; and when stocks go down, volatilities seem to go up”.

Cox (1975) proposed a call valuation model which assumed that the volatility is related, by a negative and constant relation, to the asset price. The diffusion process characterizing the Cox model takes the form

\[
\frac{dS}{S} = \mu \cdot dt + \delta \cdot S^ {\theta - 1} \cdot dz
\]

\( \mu \): the expected asset return rate

\( \delta \cdot S^{\theta - 1} \): the instantaneous standard deviation of the return rate asset, with \( \delta \) and \( \theta \) being constants

\( dz \): a Wiener standard process

The elasticity variance \( h_s \) with respect to the stock price is

\[
h_s = \left( \frac{\partial \sigma^2}{\partial S} \right) \left( \frac{S}{\sigma^2} \right) = \frac{\partial (\delta^2 \cdot S^{\theta - 2})}{\partial S} \cdot \frac{S}{\delta^2 \cdot S^{\theta - 2}} = \theta - 2
\]

It is easily seen that the model CEV will be equivalent to the BS model when \( \theta = 2 \) and that the volatility is a decreasing function of \( S_t \), as reported by Black (1975), if \( \theta < 2 \). Under the differential equation (11) and the set as assumptions in the BS framework\(^9\), Cox (1975) derived the equilibrium price formula of European call option\(^{10}\) for \( \theta < 2 \). We can clearly see that the elasticity \( h_s \) is constant

---


\(^9\) Although the variance is no longer stationary, according to the equation (11), it’s still a deterministic (constant) function of the asset price. The Cox model is a one state variable model, which is once again the asset price. Under these circumstances, it’s possible to establish a perfectly covered position using the call and its underlying asset. When applying the no arbitrage assumption, it becomes possible to obtain the call pricing formula. Moreover, this formula can be applied the risk preferences. Indeed, in a neutral risk economy, individuals require no risk premium. All assets have an expected return rate equal to the risk free interest rate.

\(^{10}\) Cox option pricing formula is developed in appendix A.
and negative. That’s why the Cox model is often called the Constant Elasticity Variance (CEV) model. The CEV European call price details are presented in the appendix.

Several empirical studies, Emanuel and Mac Beth (1982), Bates (1995) and Jones (2003), had shown that the CEV model corrects a systematic bias of the BS model which tends to overprice the in-the-money calls and underprice the out-of-the-money ones. However, when the volatility is inversely related to the asset price, high (low) level asset prices must have low (high) volatilities, so the in-the-money (out-of-the-money) calls CEV prices will be lower (higher) than the BS ones.

As the warrant life term is generally higher than one year, we should use a valuation model that allows variance to vary through time. For that reason, using CEV model to pricing warrants should be more appropriate than the use of the BS model. To calculate the CEV warrant price, we introduced 2 main modifications to the general formula set up by Cox. First, we fix $\theta = 1$. This choice will considerably simplify the Cox formula. More, several studies, Beckers (1980)$^{11}$, Lauterbach and Schultz (1990)$^{12}$, Hauser and Lauterbach (1997)$^{13}$, provide encouraging estimates of warrants and options prices when keeping $\theta = 1$. The CEV model will be transformed on what’s called “Square Root Constant Elasticity Variance” (SRCEV) model. (See the formula details in appendix). The second modification made to the CEV model is that we integrated the dilution effect as we did for the DABS model.

The modified SRCEV model is

$$W_t = \frac{n}{\lambda + m} \left[ \left( S_t + \frac{m}{n} \cdot w \right) \cdot N[q(4)] - K \cdot e^{\cdot r \cdot t} \cdot N[q(0)] \right]$$  \hspace{1cm} (13)

$q(4)$ and $q(0)$ are developed in appendix B.

**IV- Database**

The database used for this study consists on:

- Warrants prices: the end-of-the day prices for the warrants negotiated on the Jakarta Stock Exchange (JSX) during the period going from 01/02/2001 until the 12/31/2003. We count 85 call warrants. Theses prices were directly obtained from the JSX communication and public relationship department.

- Information bulletins: also obtained from the JSX communication and public relationship department. These bulletins indicate, for each warrant, the issue date, the exercise price, the expiry date. The conversion ratio is equal to 1 for all the warrants negotiated on the JSX during this period.

- Underlying shares prices: the end-of-the day shares prices. The total number of the observed prices is 29978. However, neither the database, obtained from the JSX communication and

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13 S. Hauser and B. Lauterbach (1997). “The Relative Performance of Five Alternative Warrant Pricing Models”. Financial Analysts Journal. This paper was interested in comparing the performance of five pricing warrants models traded at the American market for the period 1971 to 1980. The models used were the BS model, the DABS model, the Longstaff model for extended warrants, the SRCEV model and the Ritchken free theta model. The latter gave the lowest estimation error (3.6%) ahead of the SRCEV model (3.67%) ranked in 2nd position.
public relationship department, nor that downloaded from the web site [www.jsx.co.id](http://www.jsx.co.id) provides information about dividend distribution.

- Riskless Interest Rates: the interest rates for governmental obligations issued during 2001, 2002 and 2003. The information were downloaded from Indonesian Central Bank web site: [www.bi.go.id](http://www.bi.go.id).

Several filters were used in order to reach a sample that can be used for a reliable analysis. We eliminate warrants with incomplete information bulletin. Then, we eliminate warrants inappropriate for the pricing models used through our paper, i.e. the warrants with a “set up” exercise price following a stock split operation. We also apply a liquidity filter. We retain warrants with a liquidity level higher than 20%. The warrants prices that do not match the no arbitrage assumption are excluded from the sample. Finally, we keep the warrants that have, at least, 100 useable observations. The summarized statistics for the final sample are presented at table 1. The distribution of the observation according to the criteria of moneyness\(^{14}\) and time to expiration are presented at table 2. It’s noted that the final sample is dominated by deep out and out-of-the-money warrants, which account for 83.61% of the total observations. Regarding the time to expiration, 73.06% of the observations show a time to expiration beyond a year versus 23.7% with a time to expiration less than a year.

V- “Dilution Adjusted Black & Scholes” vs. “Black & Scholes”

We calculate the estimation error as the absolute deviation of the market warrant price from the theoretical price, reported to the warrant price. This measure illustrates the exactness with which each model fits the observed prices. The average estimation error (AEE), for the whole observations, can be presented as follows

\[
AEE = \frac{1}{n} \sum_{j=1}^{n} \left| \frac{W_{j,\text{model}} - W_{j,\text{market}}}{W_{j,\text{market}}} \right| \tag{14}
\]

Where (n) is the number of observed warrant prices across all warrants and days of the sample.

The AEE, calculated through 5460 observations, are 14.45% and 12.76% respectively for BS and DABS models. Applying the z-test, the minimum significant difference test, we find that the 2 AEE are significantly different from each other at level \(\alpha=1\%\).

Now, let calculate the AEE by warrant. Table (3) provides the results. It also provides the factors that may explain the performance of each of the 2 models.

Except for the warrant TMPO-W, the DABS model provides better estimates than BS model. This superiority can be explained by the fact that DABS model succeeded to correctly treat the dilution effect by making an explicit adjustment of the BS model, via the introduction of the dilution factor \(\frac{n}{n+m} \). An implicit adjustment for the dilution effect through the underlying asset price, as assumed by Bensoussan, Crouhy and Galai (1995), doesn’t seem very suitable for the Indonesian market. We clearly see that the highest difference, between the estimation errors of each model, was

\[^{14}\text{Moneyness is calculated as } \frac{S_t - K \cdot e^{-r\tau}}{K \cdot e^{-r\tau}} \text{ where } S_t \text{ is the underlying asset price, } K \text{ the warrant exercise price, } r \text{ the riskless interest rate and } \tau \text{ the time to expiration.}\]
recorded for the warrant that has the highest dilution factor (warrant PLAS-W with a dilution factor of 74%). On the other side, the smallest difference was recorded for the warrant with the smallest dilution factor (warrant ITTG-W with a dilution factor of 9%).

We also classify the observation into 9 subsamples using the warrant moneyness and time to expiration criteria. We made same calculus and same comparison between the 2 models for each of these categories. Results are summarized in table 4.

We clearly see that the DABS average estimation error get closer to the BS one as we move from out-of-the money warrants subsamples to in-the-money ones. Moreover, when applying the z-test, we find that the DABS estimation errors are significantly different from the BS ones (significance at level $\alpha=5\%$) only for deep out-of-the money subsamples. Such a result can be explained by the fact that these warrants are precisely those who have the highest dilution factor (see table 1).

Another finding is that, both DABS and BS models perform the worst for out and deep-out-of-the money warrants, exactly as do the classic BS option valuation formula. This is due to the fact that these 2 models still use the underlying asset price lognormality assumption with a constant volatility. Such an assumption is hardly suitable for a long life term asset as the warrant is.

**VI- the volatility Behaviour**

As shown in table 4, the AEE varies, dramatically, as we move cross subsamples. This may be the result of strong relationship between the local volatility rate and the asset price. To confirm this assumption, we can regress the implied volatility on the moneyness factor. The existence of a significant negative relation between the asset price and its volatility will, necessarily, reinforce the position of the modified SRCEV model versus the BS and DABS ones. To have a more clear idea on this issue, we apply the following regression:

$$ISD_t = \alpha_0 + \alpha_1 \left( \frac{S_t - K \cdot e^{-r \tau}}{K \cdot e^{-r \tau}} \right)_t + \varepsilon_t$$  

(15)

Where $ISD_t$ is the warrant implied standard deviation calculated at date $t$, $\left( \frac{S_t - K \cdot e^{-r \tau}}{K \cdot e^{-r \tau}} \right)_t$ is the moneyness factor.

First of all, we do the regression for the whole panel data, across all warrants. We apply three different panel regression models: Fixed Effects Model, Weighted Fixed Effects Model and Random Effects Model. The regression results are summarized at table 5.
Table 1 - DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Presence in the sample</th>
<th>Obs.</th>
<th>Dilution</th>
<th>Expiration</th>
<th>Strike Price</th>
<th>Mean «Moneyness»</th>
<th>Mean ISD</th>
<th>Mean time to expiration</th>
<th>Life Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>10/08/01 – 12/30/03</td>
<td>385</td>
<td>28.57%</td>
<td>07/19/2004</td>
<td>300</td>
<td>-0.363</td>
<td>0.4759</td>
<td>1.487</td>
<td>3 years</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>01/18/02 – 12/30/03</td>
<td>317</td>
<td>33.33%</td>
<td>01/18/2005</td>
<td>250</td>
<td>-0.261</td>
<td>0.9276</td>
<td>1.759</td>
<td>3 years</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>06/12/01 – 12/30/03</td>
<td>397</td>
<td>12.5%</td>
<td>06/07/2004</td>
<td>250</td>
<td>-0.02</td>
<td>0.3668</td>
<td>1.38</td>
<td>3 years</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>04/16/02 – 12/31/03</td>
<td>380</td>
<td>44.44%</td>
<td>11/22/2004</td>
<td>125</td>
<td>-0.894</td>
<td>1.3365</td>
<td>1.83</td>
<td>3 years</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>08/12/02 – 12/30/03</td>
<td>319</td>
<td>20%</td>
<td>08/11/2005</td>
<td>275</td>
<td>-0.455</td>
<td>0.5059</td>
<td>2.314</td>
<td>3 years</td>
</tr>
<tr>
<td>IDS-R-W</td>
<td>03/22/01 – 12/30/03</td>
<td>602</td>
<td>16.67%</td>
<td>03/21/2004</td>
<td>650</td>
<td>0.169</td>
<td>0.9304</td>
<td>1.457</td>
<td>3 years</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>11/26/01 – 12/31/03</td>
<td>490</td>
<td>9.09%</td>
<td>11/25/2004</td>
<td>150</td>
<td>-0.324</td>
<td>0.4829</td>
<td>1.964</td>
<td>3 years</td>
</tr>
<tr>
<td>KARK-W</td>
<td>07/20/01 – 12/30/03</td>
<td>575</td>
<td>48.45%</td>
<td>07/19/2004</td>
<td>125</td>
<td>-0.704</td>
<td>0.819</td>
<td>1.799</td>
<td>3 years</td>
</tr>
<tr>
<td>KREN-W</td>
<td>06/28/02 – 12/30/03</td>
<td>347</td>
<td>16.67%</td>
<td>06/28/2005</td>
<td>265</td>
<td>-0.577</td>
<td>0.5507</td>
<td>2.258</td>
<td>3 years</td>
</tr>
<tr>
<td>META-W</td>
<td>07/18/01 – 07/10/02</td>
<td>239</td>
<td>50%</td>
<td>07/17/2002</td>
<td>200</td>
<td>-0.481</td>
<td>1.139</td>
<td>0.503</td>
<td>1 year</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>03/16/01 – 12/30/03</td>
<td>585</td>
<td>74.07%</td>
<td>03/15/2004</td>
<td>200</td>
<td>-0.538</td>
<td>0.6317</td>
<td>1.473</td>
<td>3 years</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>01/08/02 – 12/31/03</td>
<td>451</td>
<td>44.44%</td>
<td>01/07/2004</td>
<td>300</td>
<td>-0.394</td>
<td>0.7342</td>
<td>1.06</td>
<td>2 years</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>01/02/02 – 12/30/03</td>
<td>466</td>
<td>20%</td>
<td>06/21/2004</td>
<td>175</td>
<td>-0.754</td>
<td>1.1501</td>
<td>1.492</td>
<td>2.5 years</td>
</tr>
</tbody>
</table>

1 The number of observations of each warrant in the sample.
2 The implied standard deviation.
Table 2 - Distribution of the Observations according to the moneyness and the time to expiration criteria

The distribution of the 5460 observations will be based on 2 criteria: the moneyness and the time to expiration. Figures in parentheses are the percentages of observations of each subsample compared to the whole sample. Deep in-the-money warrants are those with a moneyness factor > 0.5. In-the-money warrants have a moneyness factor $\in [0.05; 0.5]$. At-the-money warrants have a moneyness $\in [-0.05; 0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5; -0.05]$. Deep out-of-the-money warrants have a moneyness factor < -0.5.

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>≤ 1 year</th>
<th>&gt; 1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In the money</td>
<td>0 (0%)</td>
<td>103 (1.89%)</td>
</tr>
<tr>
<td></td>
<td>38 (0.70%)</td>
<td>408 (7.47%)</td>
</tr>
<tr>
<td>In the money</td>
<td>155 (2.83%)</td>
<td>206 (3.77%)</td>
</tr>
<tr>
<td></td>
<td>202 (3.70%)</td>
<td>1508 (27.62%)</td>
</tr>
<tr>
<td>At the money</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out of the money</td>
<td>899 (16.47%)</td>
<td>1941 (35.55%)</td>
</tr>
<tr>
<td>Deep Out of the money</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 - Comparison of Estimation Errors by Warrant

A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors.

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Observations</th>
<th>Mean Moneyness</th>
<th>Dilution</th>
<th>Error BS</th>
<th>Error DABS</th>
<th>Performance BS – DABS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>347</td>
<td>-0.363</td>
<td>28.57%</td>
<td>14.73%</td>
<td>14.28%</td>
<td>0.46%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>299</td>
<td>-0.261</td>
<td>33.33%</td>
<td>3.56%</td>
<td>3.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>360</td>
<td>-0.02</td>
<td>12.50%</td>
<td>7.15%</td>
<td>5.27%</td>
<td>1.88%</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>379</td>
<td>-0.894</td>
<td>44.44%</td>
<td>21.40%</td>
<td>18.34%</td>
<td>3.06%</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>316</td>
<td>-0.455</td>
<td>20%</td>
<td>13%</td>
<td>12.30%</td>
<td>0.70%</td>
</tr>
<tr>
<td>IDSR-W</td>
<td>601</td>
<td>0.169</td>
<td>16.67%</td>
<td>5.07%</td>
<td>4.91%</td>
<td>0.16%</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>487</td>
<td>-0.324</td>
<td>9.09%</td>
<td>11.93%</td>
<td>11.82%</td>
<td>0.11%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>570</td>
<td>-0.704</td>
<td>48.45%</td>
<td>20.80%</td>
<td>18.82%</td>
<td>1.98%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>344</td>
<td>-0.577</td>
<td>16.67%</td>
<td>15.47%</td>
<td>14.94%</td>
<td>0.53%</td>
</tr>
<tr>
<td>META-W</td>
<td>236</td>
<td>-0.481</td>
<td>50%</td>
<td>24.28%</td>
<td>22.88%</td>
<td>1.40%</td>
</tr>
</tbody>
</table>
A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor>0.5. In-the-money warrants have a moneyness factor ∈ [0.05; 0.5]. At-the-money warrants have a moneyness ∈ [0.05; 0.05]. Out-of-the-money warrants have a moneyness factor ∈ [−0.5;−0.05]. Deep out-of-the money warrants have a moneyness factor<-0.5.

<table>
<thead>
<tr>
<th></th>
<th>BS</th>
<th>DABS</th>
<th>Difference</th>
<th>Probability17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.44%</td>
<td>4.87%</td>
<td>-0.43%</td>
<td>0.2562</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.31%</td>
<td>5.26%</td>
<td>0.05%</td>
<td>0.469</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>25.58%</td>
<td>35.76%</td>
<td>-10.18%</td>
<td>0.2789</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>6.86%</td>
<td>5.71%</td>
<td>1.15%</td>
<td>0.2726</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>10.07%</td>
<td>9.92%</td>
<td>0.15%</td>
<td>0.4859</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>11.72%</td>
<td>10.71%</td>
<td>1.01%</td>
<td>0.1312</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>14.55%</td>
<td>12.99%</td>
<td>1.56%</td>
<td>0.293</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>16.2%</td>
<td>14.13%</td>
<td>2.07%</td>
<td>0.033</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>22.2%</td>
<td>18.31%</td>
<td>3.89%</td>
<td>0.0107</td>
</tr>
</tbody>
</table>

17 The null hypothesis probabilities for the z-test.
Comparative table aimed to identify the estimation results given by each regression model. The criteria used in this comparison are: the calculated (t) student, the regression linearity coefficient and the sum of residual squares. We compare the Fixed Effects Model (FEM), the weighted FEM and the Random Effects Model (REM).

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>FEM</th>
<th>weighted FEM</th>
<th>REM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.455</td>
<td>-1.331</td>
<td>-1.480</td>
</tr>
<tr>
<td>t($\alpha_1$)</td>
<td>-64.708</td>
<td>-99.205</td>
<td>-83.555</td>
</tr>
<tr>
<td>$R^2$</td>
<td>61.69%</td>
<td>78.21%</td>
<td>70.28%</td>
</tr>
<tr>
<td>$R^2_{\text{adjusted}}$</td>
<td>61.6%</td>
<td>78.15%</td>
<td>70.28%</td>
</tr>
<tr>
<td>SCR</td>
<td>506.673</td>
<td>382.446</td>
<td>393.009</td>
</tr>
</tbody>
</table>

All regression models show a strong inverse relationship between implied volatility and moneyness. The three $\alpha_i$ estimations are significantly negative at level $\alpha=1\%$. The best regression model seems to be the Weighted Fixed Effects Model providing the most significant $\alpha_i$ estimation with the highest “t” student, the best linearity and adjusted linearity coefficients and the lowest sum of squared residuals. Such a result suggests that the modified SRCEV model would be more appropriate to fit warrants prices than DABS and BS models.

According to stochastic volatility models, Wiggins (1987), Hull and White (1987), Heston (1993), the volatility varies through time without being linked to the underlying asset price by a deterministic relation. In other terms, these models allow arbitrary correlation between volatility and asset price. Such volatility behaviour, as shown by Heston (1993), produces a probability distribution of the asset returns with high kurtosis. Thus, the relation between the implied volatility and the underlying asset price can vary as the warrant is in or out-of-the-money.

To verify whether the volatility is purely stochastic or not, we repeat the regression, using the weighted Fixed Effect Model, only for in-the-money warrants. The in-the-money observations represent 12% of the whole panel (665 observations among a total of 5460 warrants prices). The regression results are summarized in table 6.

---

Although the inverse relation is more pronounced for out-of-the-money warrants, the regression has shown that such relation is maintained for in-the-money ones with an $\alpha_1$ estimation significantly negative at level $\alpha = 1\%$. Then, we can deduce that the volatility remains inversely related to the underlying asset price, as assumed by the SRCEV model, and not purely stochastic.

**VII- Modified “SRCEV” vs. “DABS”**

The AEE, calculated through the 5460 observations, are 8.95% and 12.76% respectively for modified SRCEV and DABS models. Such a result was predictable as the modified SRCEV is adapted to the 2 warrant specificities, i.e. the dilution effect and the warrant long life term, by incorporating the dilution factor in the original pricing formula and using a variable volatility. The $z$-test shows that the 2 AEE are significantly different from each others at level $\alpha = 1\%$.

Table 7 compares the AEE calculated for each warrant using the 2 models.

**Table 7 - Comparison of Estimation Errors by Warrant**

A summary table which aims to compare the performances provided, respectively, by the DABS model and the SRCEV model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors.
Now let calculate the AEE across subsamples made upon the moneyness and the time to expiration criteria. Results are summarized at table 8.

We find that the modified SRCEV estimation errors are significantly different, at level $\alpha = 5\%$, from the DABS errors, for the out-of-the-money subsamples. On the other hand, we can see that the 2 models estimation errors are not significantly different from each others, for at and in-the-money warrants. The 2 models provide an almost similar performance for these categories of warrants.

Such a result can be explained by the modified SRCEV assumption for fixed $\theta = 1$ across all the moneyness categories and for all warrants. This assumption may be suitable for out-of-the money warrants, where the modified SRCEV model had performed its best performance, but it seems that it doesn’t work so good for at and in-the-money subsamples. For these categories of warrants, there is still an inverse relation between volatility and asset price (as shown in the previous section) but the correlation between these 2 variables should be more moderate than for out-of-the money warrants. Investors tend to be very frightened face any downtrend in the market and to demonstrate a very moderate optimism when the movement becomes bullish. Such behaviour is quite remarkable response to periods of financial crises. The volatility decreases monotonically as the asset price goes upward but the rate of decrease should be less than the rate of volatility increase when asset price goes downward. Thus $\theta$ must vary as we move from out-of-the money to in-the-money subsamples.

<table>
<thead>
<tr>
<th></th>
<th>AEE</th>
<th>$%$</th>
<th>$%$</th>
<th>$%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNKO-W</td>
<td>-0.894</td>
<td>18.34%</td>
<td>14.13%</td>
<td>4.21%</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>-0.455</td>
<td>12.3%</td>
<td>8.85%</td>
<td>3.45%</td>
</tr>
<tr>
<td>IDS-R</td>
<td>0.169</td>
<td>4.91%</td>
<td>3.49%</td>
<td>1.42%</td>
</tr>
<tr>
<td>ITT-G</td>
<td>-0.324</td>
<td>11.82%</td>
<td>11.29%</td>
<td>0.53%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>-0.704</td>
<td>18.82%</td>
<td>9.81%</td>
<td>9.01%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>-0.577</td>
<td>14.94%</td>
<td>10.17%</td>
<td>4.77%</td>
</tr>
<tr>
<td>META-W</td>
<td>-0.481</td>
<td>22.876%</td>
<td>15.31%</td>
<td>7.566%</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>-0.538</td>
<td>12.72%</td>
<td>12.14%</td>
<td>0.58%</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>-0.394</td>
<td>13.89%</td>
<td>9.72%</td>
<td>4.17%</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>-0.754</td>
<td>14.42%</td>
<td>8.59%</td>
<td>5.83%</td>
</tr>
</tbody>
</table>
Table 8 - Estimation Errors classified by Moneyness and Time to expiration

A summary table which aims to compare the performances provided, respectively, by the DABS model and the SRCEV model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor>0.5. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [-0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5; -0.05]$. Deep out-of-the-money warrants have a moneyness factor<-0.5.

<table>
<thead>
<tr>
<th></th>
<th>DABS</th>
<th>SRCEV</th>
<th>Difference</th>
<th>Probability*21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.87%</td>
<td>3.98%</td>
<td>0.89%</td>
<td>0.318</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.26%</td>
<td>4.36%</td>
<td>0.9%</td>
<td>0.411</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>35.76%</td>
<td>14.76%</td>
<td>21%</td>
<td>0.116</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>9.92%</td>
<td>9.01%</td>
<td>0.91%</td>
<td>0.420</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>5.71%</td>
<td>5.54%</td>
<td>0.17%</td>
<td>0.1574</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>10.71%</td>
<td>8.26%</td>
<td>2.45%</td>
<td>0.0384</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>12.99%</td>
<td>11.03%</td>
<td>1.96%</td>
<td>0.0005</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>18.31%</td>
<td>10.89%</td>
<td>7.42%</td>
<td>0</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>14.13%</td>
<td>9.72%</td>
<td>4.41%</td>
<td>0</td>
</tr>
</tbody>
</table>

Conclusion

As expected, the modified SRCEV model led to the lowest AEE for the whole panel, for each warrant and for the different moneyness subsamples. This result can be explained by the fact that modified SRCEV is the only one of the 3 models used in our empirical study, which tries to exploit the 2 major warrant specificities. First, the modified SRCEV is explicitly adjusted for the dilution effect which should allow offsetting the negative impact that dilution could have on the quality of estimation. Also, unlike BS and DABS models, the modified SRCEV is more adapted to the warrant long life term, proposing a diffusion process with variable underlying asset price volatility. This assumption seems to be more consistent than that of constant volatility used by BS and DABS models. However, the modified SRCEV model may be subject to some criticism. We can refer, in particular, its use of a constant elasticity variance across all the underlying assets. This elasticity can even vary for the same underlying asset depending if the spot price is going upward or downward.

To remedy this deficiency, one can calculate the implied elasticity using the observed warrants prices and then calibrate the modified SRCEV model so it can be adapted for the underlying asset specificities.

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21 The null hypothesis probabilities for the z-test.
Bibliography


Appendix A - Cox Option Pricing Formula

Cox option pricing formula under the assumption of a constant elasticity variance is defined in the following manner:

\[
C_v = S \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1) \cdot G\left(\lambda \cdot (K \cdot e^{-\tau})^n, n+1 - \frac{1}{\phi}\right) - K \cdot e^{-\tau} \cdot \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1 - \frac{1}{\phi}) \cdot G\left(\lambda \cdot (K \cdot e^{-\tau})^n, n+1\right)
\]

\(\phi = 2 \theta - 2\); 
\(\lambda = 2 \frac{r}{\delta^2} \cdot \phi \cdot e^{(\theta - r)}\); 

\(\Gamma(n) = \int_0^\alpha e^{-v} \cdot v^{n-1} \cdot dv\) : the gamma function; 

\(g(x, n) = e^{-\tau} \cdot x^{n-1} \cdot \frac{1}{\Gamma(n)}\) : the density gamma function; 

\(G(a, n) = \int_0^\alpha g(x, n) \cdot dx\) : the complementary gamma distribution function.

Appendix B - SRCEV option pricing formula as proposed by Beckers (1980)

For \(v=0\) or \(v=4\),

\[
h(v) = 1 + h(h - 1) \cdot p - \frac{1}{2} \cdot h(h - 1)(2 - h)(1 - 3h) p^2 - \left[\frac{z}{(v + y)}\right]^h \\
\frac{[2h^2 \cdot p \cdot [1 - (1 - h)(1 - 3h) p^2]]^{0.5}}{[2h^2 \cdot p \cdot [1 - (1 - h)(1 - 3h) p^2]]^{0.5}},
\]

\(h(v) = 1 - \frac{2(v + y) \cdot (v + 3y)}{3(v + 2y)^2}\), 
\(p(v) = \frac{(v + 2y)}{(v + y)^2}\), 
\(y = \frac{4r \cdot (S + \frac{M}{N} \cdot W)}{\sigma^2 (1 - e^{-\tau})}\) et 
\(z = -\frac{K \cdot y}{S + \frac{M}{N} \cdot W}\).
Pricing Warrants Models: An Empirical Study of the Indonesian Market

Authors: Zouhaier BEN KHELIFA¹; Wajih ABBASSI²
Lecturer: Wajih ABBASSI

Abstract
The main issue during periods of financial crisis is to restore the investor’s confidence and attract them back to financial markets. Thus, the warrants have been a great help to relaunch many south East Asian financial markets just after the 1997 crisis, by encouraging investors to finance the restructuring of troubled companies. However, the few empirical studies which were interested in this product, was limited to the developed markets and in particular the American one. In this paper, we will try to emphasize the warrant’s specificities compared to the option ones. Then, we will seek to release the ideal model ready to provide the best pricing for this product. The comparison will relate to 3 models: the Black and Scholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and finally the modified Square Root Constant Elasticity of Variance (SRCEV) adjusted by the dilution effect. The empirical study, which is spread out over 3 years (2001–2003), will relate to an emergent market: the Jakarta Stock Exchange (JSX).

Key words: Black and Scholes, Constant elasticity variance, Dilution, Indonesian market, Moneyness, Warrant valuation.

JEL Classification: G12, G13

Theme: Financial crisis and asset pricing

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Introduction

Warrants are financial instruments issued by financial institutions and are generally negotiated on the options markets. A warrant holder has the right to buy or to sell a specific underlying asset once reached the warrant expiry at a fixed price called the strike price. At first sight, one can confuse warrant with conventional option. Both share the same principles of underlying asset, strike price and expiry date. Moreover, several papers used to valuate warrant price as same as option’s one using black and Scholes formula. However, there are two major differences between warrants and conventional options. First, the warrant lifetime is quite large than the option’s one. A warrant can have a time to expiration going up to seven years whereas an option commonly has a time to expiration not exceeding few months. Second, options are issued by individuals so an eventual exercise will only lead to the underlying asset transfer from one operator to another one. Warrants are issued by firms. Consequently, their exercise will cause the issue of additional shares and thus a firm equity dilution.

In spite of warrants success, a few papers were involved on their valuation. Moreover, most of these papers were focused on the developed markets. For this paper, we are interested on empirical validation of three valuation models for warrants issued on an emergent market. We select the Indonesian market for 2 reasons. First, because of the south East Asian financial market dynamic character this doesn’t cease attracting all kinds of investors and speculators. Also, the Indonesian warrant market recorded the highest growth rate, in term of stock exchange capitalization, compared to other Indonesian financial markets, particularly after the 1997 crisis. At those circumstances, warrants were issued to encourage reinvesting in Indonesian stocks.

We will try to compare the performances of three warrant valuation models using the data observed on the Jakarta Stock Exchange for the years 2001, 2002 and 2003. The models tested are the Black and Scholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and the modified Square Root Constant Elasticity Variance model (modified SRCEV) inspired from Cox (1975) Constant Elasticity Variance Model (CEV).

This paper will be structured as follows. The first section introduces the BS and DABS models. The second introduces the CEV model and the necessary modifications, in order to make the Cox Model more suitable for warrants valuation. The third section introduces the statistical summaries of the data used through our empirical study. The fourth section compares the BS and DABS valuation performances. The fifth deals about the underlying assets volatility behaviour. The sixth section compares the modified SRCEV and DABS valuation performances. Finally, we conclude by a brief summary of the main results and some suggestion for future research.

I- Valuating warrants with BS and DABS models

1. **DABS model: Galai and Schneller approach (1978)**

Like a conventional European call, the warrant is the right to buy, at the strike price, an underlying asset at an expiration date. However, warrants are issued by companies so that their exercise will lead to an issue of additional shares and, therefore, a dilution of the issuing company equities. We try to emphasize the relation between the issuing firm value and the warrant price.

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3 Warrants have appeared for the first time on the Jakarta Stock Exchange dated July 13, 1995 and were issued by the bank TIARA ASIA.

4 J. Cox (1975). “Notes on option pricing: Constant Elasticity Variance Diffusions”. Working Paper, Stanford University. Through this paper, Cox has established a new model for pricing options assuming that volatility, is no longer constant, but moves inversely to the underlying asset price.

Let consider a firm (A) with 100% of equities and (n) shares. Suppose that this firm will be liquidated at date (T) with a stochastic future value $V_T$. Each share value, at that date, will be

$$S_T = \frac{V_T}{n}.$$  

Now let consider a second firm (B), identical to the first, that issues (m) warrants at t=0, with strike price $K$ and expiration date $T$. Each exercised warrant allows its holder to possess a new issuing firm share. So, the conversion ratio, noted $\lambda$, is equal to 1. If warrants are exercised at expiration date, the firm B value will be higher than the firm (A) one.

$$V_T^* = V_T + m \cdot K$$  \hspace{1cm} (1)

The share value of the firm (B) at time $T$, noted $S_T^*$, will be equal to

$$S_T^* = \frac{V_T + m \cdot K}{n + m}$$  \hspace{1cm} (2)

The warrant will be exercised at expiration date, only if the underlying stock price exceeds the warrant strike price.

$$S_T^* > K \Rightarrow \frac{V_T + m \cdot K}{n + m} > K$$  \hspace{1cm} (3)

In other hand, $S_T = \frac{V_T}{n}$, so

$$S_T^* = \frac{n \cdot S_T + m \cdot K}{n + m} > K$$  \hspace{1cm} (4)

Now, let try to infer the warrant price using the BS European call pricing formula. Consider a European call with the firm (A) share as an underlying asset. At expiration date, the call value will be equal to Max ($0 ; S_T - K$). At the same date, the warrant value will be equal to Max ($0 ; S_T^* - K$).

$$S_T^* - K = \frac{n \cdot S_T + m \cdot K}{n + m} - K = \frac{n}{n + m} (S_T - K)$$  \hspace{1cm} (5)

If $S_T > K$, both the call and the warrant will be exercised, so

$$W_T = \frac{n}{n + m} \cdot C_T$$  \hspace{1cm} (6)

If $S_T < K$, the call and the warrant will be abandoned and will have a null value.

We conclude that the warrant and call prices are perfectly correlated. In an efficient market, and with the no arbitrage assumption, 2 financial assets offering 2 perfectly correlated outputs will have their prices correlated with the same coefficient at any time $t \in [0; T]$. So

$$W_t = \frac{n}{n + m} \cdot C_t$$  \hspace{1cm} (7)

As we substitute $C_t$ by the BS formula, the warrant value can be written as

$$W_t = \frac{n}{n + m} \left[ S_t \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \right]$$  \hspace{1cm} (8)
Consider a conversion ratio $\lambda$, the DABS warrant pricing formula as presented Galai and Schneller is

$$W_t = \frac{n}{n + m} \cdot \left[ S_t + \frac{m \cdot w}{n} \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) \right]$$  \hspace{1cm} (9)$$

$W_t$: the warrant price at time (t)

$S_t$: the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two bonds whose maturities straddle the warrant expiration date.

$\tau$: the warrant time to expiration

$$d_1 = \ln \left( \frac{S_t + \frac{m \cdot w}{n}}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau$$

$$d_2 = d_1 - \sigma \cdot \sqrt{\tau}$$

$m$: number of issued warrants

$n$: number of underlying shares

$w$: the warrant price at the issue date

$\lambda$: Conversion ratio, i.e. for each warrant exercised, $\lambda$ new underlying shares are created.

2. **BS model: Bensoussan, Crouhy and Galai approach (1995)**

This approach assumes that the underlying asset price reflects the dilution effect. The current shareholders anticipate the effect of an eventual dilution at the issue announcement. At that date, there would be a jump in the underlying asset price. That’s why; Bensoussan, Crouhy and Galai consider that there is no need to make modifications to the BS model when it comes to evaluating warrants because the dilution effect is directly included into the underlying asset price. The warrant theoretical price will be

$$W_t = S_t \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2)$$  \hspace{1cm} (10)$$

$W_t$: the warrant price at time (t)

$S_t$: the underlying asset price at time (t)

$K$: the warrant strike price

$r$: riskless interest rate obtained by interpolating the rate for the two T-bills whose maturities straddle the warrant expiration date.

---

6 Indeed, whenever a warrant holder decides to exercise, the issuing company will be required to create $\lambda$ new shares for him.

\(\tau\): the warrant time to expiration

\[
\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau
\]

\[
d_1 = \frac{d_1 - \sigma \cdot \sqrt{\tau}}{\sigma \cdot \sqrt{\tau}}
\]

\(N(\cdot):\) the cumulative normal distribution

**II- The modified Square Root Constant Elasticity Variance model**

An extremely practical consequence of the underlying asset price lognormality assumption is that the historical variance can be used to predict the future volatility and thus to evaluate options. Unfortunately, empirical had proven that neither the historical nor the implied volatility can give a future variance estimate which is unique and constant over time. Indeed, there are few chances that the historical variance calculated over the five last years is equal to the historical variance calculated over the last six months. There is considerable evidence in the literature indicating that stock returns are heteroscedastic, with a probability distribution showing a negative skewness. Black (1976)\(^8\) writes “I have believed for long time that stock returns are related to volatility changes. When stocks go up, volatilities seem to go down; and when stocks go down, volatilities seem to go up”.

Cox (1975) proposed a call valuation model which assumed that the volatility is related, by a negative and constant relation, to the asset price. The diffusion process characterizing the Cox model takes the form

\[
\frac{dS}{S} = \mu \cdot dt + \delta \cdot S^{\frac{\theta}{2}} \cdot dz
\]

\(\mu\): the expected asset return rate

\(\delta \cdot S^{\frac{\theta}{2}}\): the instantaneous standard deviation of the return rate asset, with \(\delta\) and \(\theta\) being constants

\(dz\): a Wiener standard process

The elasticity variance \(h_s\) with respect to the stock price is

\[
h_s = \frac{\theta}{\sigma^2} = \frac{\partial^2 \frac{S}{\sigma^2}}{\partial S^2} = \frac{\partial S^{\frac{\theta-2}{2}}}{\partial S} = \theta - 2
\]

It is easily seen that the model CEV will be equivalent to the BS model when \(\theta = 2\) and that the volatility is a decreasing function of \(S\), as reported by Black (1975), if \(\theta < 2\). Under the differential equation (11) and the set as assumptions in the BS framework\(^9\), Cox (1975) derived the equilibrium price formula of European call option\(^{10}\) for \(\theta < 2\). We can clearly see that the elasticity \(h_s\) is constant

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\(^9\) Although the variance is no longer stationary, according to the equation (11), it’s still a deterministic (constant) function of the asset price. The Cox model is a one state variable model, which is once again the asset price. Under these circumstances, it’s possible to establish a perfectly covered position using the call and its underlying asset. When applying the no arbitrage assumption, it becomes possible to obtain the call pricing formula. Moreover, this formula can be applied the risk preferences. Indeed, in a neutral risk economy, individuals require no risk premium. All assets have an expected return rate equal to the risk free interest rate.

\(^{10}\) Cox option pricing formula is developed in appendix A.
and negative. That’s why the Cox model is often called the Constant Elasticity Variance (CEV) model. The CEV European call price details are presented in the appendix.

Several empirical studies, Emanuel and Mac Beth (1982), Bates (1995) and Jones (2003), had shown that the CEV model corrects a systematic bias of the BS model which tends to overprice the in-the-money calls and underprice the out-of-the-money ones. However, when the volatility is inversely related to the asset price, high (low) level asset prices must have low (high) volatilities, so the in-the-money (out-of-the money) calls CEV prices will be lower (higher) than the BS ones.

As the warrant life term is generally higher than one year, we should use a valuation model that allows variance to vary through time. For that reason, using CEV model to pricing warrants should be more appropriate than the use of the BS model. To calculate the CEV warrant price, we introduced 2 main modifications to the general formula set up by Cox. First, we fix $\theta = 1$. This choice will considerably simplify the Cox formula. More, several studies, Beckers (1980)$^{11}$, Lauterbach and Schultz (1990)$^{12}$, Hauser and Lauterbach (1997)$^{13}$, provide encouraging estimates of warrants and options prices when keeping $\theta = 1$. The CEV model will be transformed on what’s called “Square Root Constant Elasticity Variance” (SRCEV) model. (See the formula details in appendix). The second modification made to the CEV model is that we integrated the dilution effect as we did for the DABS model.

The modified SRCEV model is

$$W_t = \frac{n}{\lambda + m} \left[ (S_t + m \cdot w) \cdot N[q(4)] - K \cdot e^{\sigma \cdot \tau} \cdot N[q(0)] \right] \quad (13)$$

$q(4)$ and $q(0)$ are developed in appendix B.

**IV- Database**

The database used for this study consists on:

- Warrants prices: the end-of-the day prices for the warrants negotiated on the Jakarta Stock Exchange (JSX) during the period going from 01/02/2001 until the 12/31/2003. We count 85 call warrants. Theses prices were directly obtained from the JSX communication and public relationship department.

- Information bulletins: also obtained from the JSX communication and public relationship department. These bulletins indicate, for each warrant, the issue date, the exercise price, the expiry date. The conversion ratio is equal to 1 for all the warrants negotiated on the JSX during this period.

- Underlying shares prices: the end-of-the day shares prices. The total number of the observed prices is 29978. However, neither the database, obtained from the JSX communication and

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$^{12}$ B. Lauterbach and P. Schultz. “Pricing Warrants: An Empirical Study of the Black Scholes Model and Its Alternatives”. Journal of Finance (1990). This paper, considered as the first one that dealt, in an exhaustive manner, the issue of warrant pricing using the DABS and the SRCEV models.

$^{13}$ S. Hauser and B. Lauterbach (1997). “The Relative Performance of Five Alternative Warrant Pricing Models”. Financial Analysts Journal. This paper was interested in comparing the performance of five pricing warrants models traded at the American market for the period 1971 to 1980. The models used were the BS model, the DABS model, the Longstaff model for extended warrants, the SRCEV model and the Ritchken free theta model. The latter gave the lowest estimation error (3.6%) ahead of the SRCEV model (3.67%) ranked in 2nd position.
public relationship department, nor that downloaded from the web site www.jsx.co.id provides information about dividend distribution.

- Riskless Interest Rates: the interest rates for governmental obligations issued during 2001, 2002 and 2003. The information were downloaded from Indonesian Central Bank web site: www.bi.go.id.

Several filters were used in order to reach a sample that can be used far a reliable analysis. We eliminate warrants with incomplete information bulletin. Then, we eliminate warrants inappropriate for the pricing models used through our paper, i.e. the warrants with a “set up” exercise price following a stock split operation. We also apply a liquidity filter. We retain warrants with a liquidity level higher than 20%. The warrants prices that do not match the no arbitrage assumption are excluded from the sample. Finally, we keep the warrants that have, at least, 100 useable observations. The summarized statistics for the final sample are presented at table 1.

The distribution of the observation according to the criteria of moneyness\(^{14}\) and time to expiration are presented at table 2. It’s noted that the final sample is dominated by deep out and out-of-the-money warrants, which account for 83.61% of the total observations. Regarding the time to expiration, 73.06% of the observations show a time to expiration beyond a year versus 23.7% with a time to expiration less than a year.

V- “Dilution Adjusted Black & Scholes” vs. “Black & Scholes”

We calculate the estimation error as the absolute deviation of the market warrant price from the theoretical price, reported to the warrant price. This measure illustrates the exactness with which each model fits the observed prices. The average estimation error (AEE), for the whole observations, can be presented as follows

\[
AEE = \frac{1}{n} \cdot \sum_{j=1}^{n} \left| \frac{W_{j,\text{model}} - W_{j,\text{market}}}{W_{j,\text{market}}} \right|
\]

Where (n) is the number of observed warrant prices across all warrants and days of the sample.

The AEE, calculated through 5460 observations, are 14.45% and 12.76% respectively for BS and DABS models. Applying the z-test, the minimum significant difference test, we find that the 2 AEE are significantly different from each other at level \(\alpha=1\%\).

Now, let calculate the AEE by warrant. Table (3) provides the results. It also provides the factors that may explain the performance of each of the 2 models.

Except for the warrant TMPO-W, the DABS model provides better estimates than BS model. This superiority can be explained by the fact that DABS model succeeded to correctly treat the dilution effect by making an explicit adjustment of the BS model, via the introduction of the dilution factor \(\frac{n}{n + m}\). An implicit adjustment for the dilution effect through the underlying asset price, as assumed by Bensoussan, Crouhy and Galai (1995), doesn’t seem very suitable for the Indonesian market. We clearly see that the highest difference, between the estimation errors of each model, was

\[\text{Moneyness is calculated as } \left( \frac{S_t - K \cdot e^{-r \tau}}{K \cdot e^{-r \tau}} \right) \text{ where } S_t \text{ is the underlying asset price, } K \text{ the warrant exercise price, } r \text{ the riskless interest rate and } \tau \text{ the time to expiration.}\]
recorded for the warrant that has the highest dilution factor (warrant PLAS-W with a dilution factor of 74%). On the other side, the smallest difference was recorded for the warrant with the smallest dilution factor (warrant ITTG-W with a dilution factor of 9%).

We also classify the observation into 9 subsamples using the warrant moneyness and time to expiration criteria. We made same calculus and same comparison between the 2 models for each of these categories. Results are summarized in table 4.

We clearly see that the DABS average estimation error get closer to the BS one as we move from out-of-the money warrants subsamples to in-the-money ones. Moreover, when applying the z-test, we find that the DABS estimation errors are significantly different from the BS ones (significance at level \( \alpha = 5\% \)) only for deep out-of-the money subsamples. Such a result can be explained by the fact that these warrants are precisely those who have the highest dilution factor (see table 1).

Another finding is that, both DABS and BS models perform the worst for out and deep-out-of-the money warrants, exactly as do the classic BS option valuation formula. This is due to the fact that these 2 models still use the underlying asset price lognormality assumption with a constant volatility. Such an assumption is hardly suitable for a long life term asset as the warrant is.

**VI- the volatility Behaviour**

As shown in table 4, the AEE varies, dramatically, as we move cross subsamples. This may be the result of strong relationship between the local volatility rate and the asset price. To confirm this assumption, we can regress the implied volatility on the moneyness factor. The existence of a significant negative relation between the asset price and its volatility will, necessarily, reinforce the position of the modified SRCEV model versus the BS and DABS ones. To have a more clear idea on this issue, we apply the following regression:

\[
ISD_t = \alpha_0 + \alpha_1 \left( \frac{S_t - K \cdot e^{-rt}}{K \cdot e^{-rt}} \right) + \varepsilon_t \quad (15)
\]

Where \( ISD_t \) is the warrant implied standard deviation calculated at date \( t \). \( \left( \frac{S_t - K \cdot e^{-rt}}{K \cdot e^{-rt}} \right) \) is the moneyness factor.

First of all, we do the regression for the whole panel data, across all warrants. We apply three different panel regression models: Fixed Effects Model, Weighted Fixed Effects Model and Random Effects Model. The regression results are summarized at table 5.
### Table 1 - DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Presence in the sample</th>
<th>Obs.</th>
<th>Dilution</th>
<th>Expiration</th>
<th>Strike Price</th>
<th>Mean «Moneyness»</th>
<th>Mean ISD</th>
<th>Mean time to expiration</th>
<th>Life Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>10/08/01 – 12/30/03</td>
<td>385</td>
<td>28.57%</td>
<td>07/19/2004</td>
<td>300</td>
<td>-0.363</td>
<td>0.4759</td>
<td>1.487</td>
<td>3 years</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>01/18/02 – 12/30/03</td>
<td>317</td>
<td>33.33%</td>
<td>01/18/2005</td>
<td>250</td>
<td>-0.261</td>
<td>0.9276</td>
<td>1.759</td>
<td>3 years</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>06/12/01 – 12/30/03</td>
<td>397</td>
<td>12.5%</td>
<td>06/07/2004</td>
<td>250</td>
<td>-0.02</td>
<td>0.3668</td>
<td>1.38</td>
<td>3 years</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>04/16/02 – 12/31/03</td>
<td>380</td>
<td>44.44%</td>
<td>11/22/2004</td>
<td>125</td>
<td>-0.894</td>
<td>1.3365</td>
<td>1.83</td>
<td>3 years</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>08/12/02 – 12/30/03</td>
<td>319</td>
<td>20%</td>
<td>08/11/2005</td>
<td>275</td>
<td>-0.455</td>
<td>0.5059</td>
<td>2.314</td>
<td>3 years</td>
</tr>
<tr>
<td>IDS-R-W</td>
<td>03/22/01 – 12/30/03</td>
<td>602</td>
<td>16.67%</td>
<td>03/21/2004</td>
<td>650</td>
<td>0.169</td>
<td>0.9304</td>
<td>1.457</td>
<td>3 years</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>11/26/01 – 12/31/03</td>
<td>490</td>
<td>9.09%</td>
<td>11/25/2004</td>
<td>150</td>
<td>-0.324</td>
<td>0.4829</td>
<td>1.964</td>
<td>3 years</td>
</tr>
<tr>
<td>KARK-W</td>
<td>07/20/01 – 12/30/03</td>
<td>575</td>
<td>48.45%</td>
<td>07/19/2004</td>
<td>125</td>
<td>-0.704</td>
<td>0.819</td>
<td>1.799</td>
<td>3 years</td>
</tr>
<tr>
<td>KREN-W</td>
<td>06/28/02 – 12/30/03</td>
<td>347</td>
<td>16.67%</td>
<td>06/28/2005</td>
<td>265</td>
<td>-0.577</td>
<td>0.5507</td>
<td>2.258</td>
<td>3 years</td>
</tr>
<tr>
<td>META-W</td>
<td>07/18/01 – 07/10/02</td>
<td>239</td>
<td>50%</td>
<td>07/17/2002</td>
<td>200</td>
<td>-0.481</td>
<td>1.139</td>
<td>0.503</td>
<td>1 year</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>03/16/01 – 12/30/03</td>
<td>585</td>
<td>74.07%</td>
<td>03/15/2004</td>
<td>200</td>
<td>-0.538</td>
<td>0.6317</td>
<td>1.473</td>
<td>3 years</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>01/08/02 – 12/31/03</td>
<td>451</td>
<td>44.44%</td>
<td>01/07/2004</td>
<td>300</td>
<td>-0.394</td>
<td>0.7342</td>
<td>1.06</td>
<td>2 years</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>01/02/02 – 12/30/03</td>
<td>466</td>
<td>20%</td>
<td>06/21/2004</td>
<td>175</td>
<td>-0.754</td>
<td>1.1501</td>
<td>1.492</td>
<td>2.5 years</td>
</tr>
</tbody>
</table>

1 The number of observations of each warrant in the sample.
2 The implied standard deviation.
Table 2 - Distribution of the Observations according to the moneyness and the time to expiration criteria

The distribution of the 5460 observations will be based on 2 criteria: the moneyness and the time to expiration. Figures in parentheses are the percentages of observations of each subsample compared to the whole sample. Deep in-the-money warrants are those with a moneyness factor > 0.5. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [-0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5;-0.05]$. Deep out-of-the-money warrants have a moneyness factor < -0.5.

<table>
<thead>
<tr>
<th></th>
<th>≤ 1 year</th>
<th>&gt; 1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deep In the money</strong></td>
<td>0 (0%)</td>
<td>103 (1.89%)</td>
</tr>
<tr>
<td><strong>In the money</strong></td>
<td>38 (0.70%)</td>
<td>408 (7.47%)</td>
</tr>
<tr>
<td><strong>At the money</strong></td>
<td>155 (2.83%)</td>
<td>206 (3.77%)</td>
</tr>
<tr>
<td><strong>Out of the money</strong></td>
<td>202 (3.70%)</td>
<td>1508 (27.62%)</td>
</tr>
<tr>
<td><strong>Deep Out of the money</strong></td>
<td>899 (16.47%)</td>
<td>1941 (35.55%)</td>
</tr>
</tbody>
</table>

Table 3 - Comparison of Estimation Errors by Warrant

A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors.

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Observations</th>
<th>Mean Moneyness</th>
<th>Dilution</th>
<th>Error BS</th>
<th>Error DABS</th>
<th>Performance BS – DABS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>347</td>
<td>-0.363</td>
<td>28.57%</td>
<td>14.73%</td>
<td>14.28%</td>
<td>0.46%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>299</td>
<td>-0.261</td>
<td>33.33%</td>
<td>3.56%</td>
<td>3.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>360</td>
<td>-0.02</td>
<td>12.50%</td>
<td>7.15%</td>
<td>5.27%</td>
<td>1.88%</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>379</td>
<td>-0.894</td>
<td>44.44%</td>
<td>21.40%</td>
<td>18.34%</td>
<td>3.06%</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>316</td>
<td>-0.455</td>
<td>20%</td>
<td>13%</td>
<td>12.30%</td>
<td>0.70%</td>
</tr>
<tr>
<td>IDSJ-W</td>
<td>601</td>
<td>0.169</td>
<td>16.67%</td>
<td>5.07%</td>
<td>4.91%</td>
<td>0.16%</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>487</td>
<td>0.324</td>
<td>9.09%</td>
<td>11.93%</td>
<td>11.82%</td>
<td>0.11%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>570</td>
<td>-0.704</td>
<td>48.45%</td>
<td>20.80%</td>
<td>18.82%</td>
<td>1.98%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>344</td>
<td>-0.577</td>
<td>16.67%</td>
<td>15.47%</td>
<td>14.94%</td>
<td>0.53%</td>
</tr>
<tr>
<td>META-W</td>
<td>236</td>
<td>-0.481</td>
<td>50%</td>
<td>24.28%</td>
<td>22.88%</td>
<td>1.40%</td>
</tr>
</tbody>
</table>
Table 4 - Estimation Errors classified by Moneyness and Time to expiration

A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor $> 0.5$. In-the-money warrants have a moneyness factor $\in [0.05; 0.5]$. At-the-money warrants have a moneyness $\in [-0.05; 0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5; -0.05]$. Deep out-of-the money warrants have a moneyness factor $<-0.5$.

<table>
<thead>
<tr>
<th></th>
<th>BS</th>
<th>DABS</th>
<th>Difference</th>
<th>Probability¹⁷</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.44%</td>
<td>4.87%</td>
<td>-0.43 %</td>
<td>0.2562</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.31%</td>
<td>5.26%</td>
<td>0.05 %</td>
<td>0.469</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>25.58%</td>
<td>35.76%</td>
<td>-10.18 %</td>
<td>0.2789</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>6.86%</td>
<td>5.71%</td>
<td>1.15 %</td>
<td>0.2726</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>10.07%</td>
<td>9.92%</td>
<td>0.15 %</td>
<td>0.4859</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>11.72%</td>
<td>10.71%</td>
<td>1.01 %</td>
<td>0.1312</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>14.55%</td>
<td>12.99%</td>
<td>1.56 %</td>
<td>0.293</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>16.2%</td>
<td>14.13%</td>
<td>2.07 %</td>
<td>0.033</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>22.2%</td>
<td>18.31%</td>
<td>3.89 %</td>
<td>0.0107</td>
</tr>
</tbody>
</table>

¹⁷ The null hypothesis probabilities for the z-test.
Comparative table aimed to identify the estimation results given by each regression model. The criteria used in this comparison are: the calculated (t) student, the regression linearity coefficient and the sum of residual squares. We compare the Fixed Effects Model (FEM), the weighted FEM and the Random Effects Model (REM).

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>FEM</th>
<th>weighted FEM</th>
<th>REM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.455</td>
<td>-1.331</td>
<td>-1.480</td>
</tr>
<tr>
<td>$t(\alpha_1)$</td>
<td>-64.708</td>
<td>-99.205</td>
<td>-83.555</td>
</tr>
<tr>
<td>$R^2$</td>
<td>61.69%</td>
<td>78.21%</td>
<td>70.28%</td>
</tr>
<tr>
<td>$R^2_{\text{adjusted}}$</td>
<td>61.6%</td>
<td>78.15%</td>
<td>70.28%</td>
</tr>
<tr>
<td>SCR</td>
<td>506.673</td>
<td>382.446</td>
<td>393.009</td>
</tr>
</tbody>
</table>

All regression models show a strong inverse relationship between implied volatility and moneyness. The three $\alpha_i$ estimations are significantly negative at level $\alpha=1\%$. The best regression model seems to be the Weighted Fixed Effects Model providing the most significant $\alpha_i$ estimation with the highest “t” student, the best linearity and adjusted linearity coefficients and the lowest sum of squared residuals. Such a result suggests that the modified SRCEV model would be more appropriate to fit warrants prices than DABS and BS models.

According to stochastic volatility models, Wiggins (1987)\textsuperscript{18}, Hull and White (1987)\textsuperscript{19}, Heston (1993)\textsuperscript{20}, the volatility varies through time without being linked to the underlying asset price by a deterministic relation. In other terms, these models allow arbitrary correlation between volatility and asset price. Such volatility behaviour, as shown by Heston (1993), produces a probability distribution of the asset returns with high kurtosis. Thus, the relation between the implied volatility and the underlying asset price can vary as the warrant is in or out-of-the-money.

To verify whether the volatility is purely stochastic or not, we repeat the regression, using the weighted Fixed Effect Model, only for in-the-money warrants. The in-the-money observations represent 12% of the whole panel (665 observations among a total of 5460 warrants prices). The regression results are summarized in table 6.

---


Table 6 - Regression results for in-the-money observations

<table>
<thead>
<tr>
<th>Warrant</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$R^2 = 93.81%$</th>
<th>$R^2_{\text{adjusted}} = 93.72%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>0.1442</td>
<td>-0.1742</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ANTA-W</td>
<td>0.3029</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCAP-W</td>
<td>0.2636</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IDS-W</td>
<td>0.6177</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITT-W</td>
<td>0.2629</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KARK-W</td>
<td>0.1735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>META-W</td>
<td>0.1753</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLAS-W</td>
<td>0.1245</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMPO-W</td>
<td>0.1483</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although the inverse relation is more pronounced for out-of-the-money warrants, the regression has shown that such relation is maintained for in-the-money ones with an $\alpha_1$ estimation significantly negative at level $\alpha = 1\%$. Then, we can deduce that the volatility remains inversely related to the underlying asset price, as assumed by the SRCEV model, and not purely stochastic.

VII- Modified “SRCEV” vs. “DABS”

The AEE, calculated through the 5460 observations, are 8.95% and 12.76% respectively for modified SRCEV and DABS models. Such a result was predictable as the modified SRCEV is adapted to the 2 warrant specificities, i.e. the dilution effect and the warrant long life term, by incorporating the dilution factor in the original pricing formula and using a variable volatility. The $z$-test shows that the 2 AEE are significantly different from each others at level $\alpha = 1\%$.

Table 7 compares the AEE calculated for each warrant using the 2 models.

Table 7 - Comparison of Estimation Errors by Warrant

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Average Moneyness</th>
<th>Error DABS</th>
<th>Error SRCEV</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>-0.363</td>
<td>14.275%</td>
<td>8.20%</td>
<td>6.075%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>-0.261</td>
<td>3.33%</td>
<td>0.47%</td>
<td>2.86%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>-0.02</td>
<td>5.27%</td>
<td>5.27%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>
Now let calculate the AEE across subsamples made upon the moneyness and the time to expiration criteria. Results are summarized at table 8.

We find that the modified SRCEV estimation errors are significantly different, at level $\alpha = 5\%$, from the DABS errors, for the out-of-the money subsamples. On the other hand, we can see that the 2 models estimation errors are not significantly different from each others, for at and in-the-money warrants. The 2 models provide an almost similar performance for these categories of warrants.

Such a result can be explained by the modified SRCEV assumption for fixed $\theta = 1$ across all the moneyness categories and for all warrants. This assumption may be suitable for out-of-the money warrants, where the modified SRCEV model had performed its best performance, but it seems that it doesn’t work so good for at and in-the-money subsamples. For these categories of warrants, there is still an inverse relation between volatility and asset price (as shown in the previous section) but the correlation between these 2 variables should be more moderate than for out-of-the money warrants. Investors tend to be very frightened face any downtrend in the market and to demonstrate a very moderate optimism when the movement becomes bullish. Such behaviour is quite remarkable response to periods of financial crises. The volatility decreases monotonically as the asset price goes upward but the rate of decrease should be less than the rate of volatility increase when asset price goes downward. Thus $\theta$ must vary as we move from out-of-the money to in-the-money subsamples.
A summary table which aims to compare the performances provided, respectively, by the DABS model and the SRCEV model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor>0.5. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [-0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5; -0.05]$. Deep out-of-the-money warrants have a moneyness factor< -0.5.

<table>
<thead>
<tr>
<th></th>
<th>DABS</th>
<th>SRCEV</th>
<th>Difference</th>
<th>Probability$^{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.87 %</td>
<td>3.98 %</td>
<td>0.89 %</td>
<td>0.318</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.26 %</td>
<td>4.36 %</td>
<td>0.9 %</td>
<td>0.411</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>35.76 %</td>
<td>14.76 %</td>
<td>21 %</td>
<td>0.116</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>9.92 %</td>
<td>9.01 %</td>
<td>0.91 %</td>
<td>0.420</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>5.71 %</td>
<td>5.54 %</td>
<td>0.17 %</td>
<td>0.1574</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>10.71 %</td>
<td>8.26 %</td>
<td>2.45 %</td>
<td>0.0384</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>12.99 %</td>
<td>11.03 %</td>
<td>1.96 %</td>
<td>0.0005</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>18.31 %</td>
<td>10.89 %</td>
<td>7.42 %</td>
<td>0</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>14.13 %</td>
<td>9.72 %</td>
<td>4.41 %</td>
<td>0</td>
</tr>
</tbody>
</table>

Conclusion

As expected, the modified SRCEV model led to the lowest AEE for the whole panel, for each warrant and for the different moneyness subsamples. This result can be explained by the fact that modified SRCEV is the only one of the 3 models used in our empirical study, which tries to exploit the 2 major warrant specificities.

First, the modified SRCEV is explicitly adjusted for the dilution effect which should allow offsetting the negative impact that dilution could have on the quality of estimation.

Also, unlike BS and DABS models, the modified SRCEV is more adapted to the warrant long life term, proposing a diffusion process with variable underlying asset price volatility. This assumption seems to be more consistent than that of constant volatility used by BS and DABS models.

However, the modified SRCEV model may be subject to some criticism. We can refer, in particular, its use of a constant elasticity variance across all the underlying assets. This elasticity can even vary for the same underlying asset depending if the spot price is going upward or downward.

To remedy this deficiency, one can calculate the implied elasticity using the observed warrants prices and then calibrate the modified SRCEV model so it can be adapted for the underlying asset specificities.

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$^{21}$ The null hypothesis probabilities for the z-test.
Bibliography


Appendix A - Cox Option Pricing Formula

Cox option pricing formula under the assumption of a constant elasticity variance is defined in the following manner:

\[
C_c = S \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1) \cdot G(\lambda \cdot (K \cdot e^{-r \tau})^n, n+1 - \frac{1}{\phi}) - K \cdot e^{-r \tau} \sum_{n=0}^{\infty} g(\lambda \cdot S^n, n+1 - \frac{1}{\phi}) \cdot G(\lambda \cdot (K \cdot e^{-r \tau})^n, n+1)
\]

\(C_c\) is a European call price with a strike price \(K\) and a time to expiration \(\tau = (T - t)\);
\(\phi = 2 \theta - 2\);
\(\lambda = 2 \frac{r}{\delta^2} \cdot \phi \cdot e^{(r \phi - 1)}\);

\(\Gamma(n) = \int_0^\infty e^{-v} \cdot v^{n-1} \cdot dv\): the gamma function;

\(g(x,n) = e^{-x} \cdot x^{n-1} \cdot \frac{1}{\Gamma(n)}\): the density gamma function;

\(G(a,n) = \int_0^\alpha g(x,n) \cdot dx\): the complementary gamma distribution function.

Appendix B - SRCEV option pricing formula as proposed by Beckers (1980)

For \(\nu=0\) or \(\nu=4\),

\[
(v) = \frac{1 + h(h - 1) \cdot p - \frac{1}{2} \cdot h(h - 1)(2 - h)(1 - 3h) p^2 - [\frac{z}{(v + y)^\nu}]^h}{[2h^2 \cdot p \cdot [1 - (1 - h)(1 - 3h)p]]^{0.5}},
\]

\(h(v) = 1 - \frac{2(v + y) \cdot (v + 3y)}{3(v + 2y)^2}\), \(p(v) = \frac{(v + 2y)}{(v + y)^2}\), \(y = \frac{4r \cdot (S + \frac{M}{N} \cdot W)}{\sigma^2 (1 - e^{-r \tau})}\) et \(z = -\frac{K \cdot y}{S + \frac{M}{N} \cdot W}\).
Pricing Warrants Models: An Empirical Study of the Indonesian Market

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Abstract

The main issue during periods of financial crisis is to restore the investor’s confidence and attract them back to financial markets. Thus, the warrants have been a great help to relaunch many south East Asian financial markets just after the 1997 crisis, by encouraging investors to finance the restructuring of troubled companies. However, the few empirical studies which were interested in this product, was limited to the developed markets and in particular the American one. In this paper, we will try to emphasize the warrant’s specificities compared to the option ones. Then, we will seek to release the ideal model ready to provide the best pricing for this product. The comparison will relate to 3 models: the Black and Sholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and finally the modified Square Root Constant Elasticity of Variance (SRCEV) adjusted by the dilution effect. The empirical study, which is spread out over 3 years (2001–2003), will relate to an emergent market: the Jakarta Stock Exchange (JSX).

Key words: Black and Scholes, Constant elasticity variance, Dilution, Indonesian market, Moneyness, Warrant valuation.

JEL Classification: G12, G13

Theme: Financial crisis and asset pricing

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Introduction

Warrants are financial instruments issued by financial institutions and are generally negotiated on the options markets. A warrant holder has the right to buy or to sell a specific underlying asset once reached the warrant expiry at a fixed price called the strike price. At first sight, one can confuse warrant with conventional option. Both share the same principles of underlying asset, strike price and expiry date. Moreover, several papers used to valuate warrant price as same as option’s one using black and Scholes formula. However, there are two major differences between warrants and conventional options. First, the warrant lifetime is quite large than the option’s one. A warrant can have a time to expiration going up to seven years whereas an option commonly has a time to expiration not exceeding few months. Second, options are issued by individuals so an eventual exercise will only lead to the underlying asset transfer from one operator to another one. Warrants are issued by firms. Consequently, their exercise will cause the issue of additional shares and thus a firm equity dilution.

In spite of warrants success, a few papers were involved on their valuation. Moreover, most of these papers were focused on the developed markets. For this paper, we are interested on empirical validation of three valuation models for warrants issued on an emergent market. We select the Indonesian market\(^3\) for 2 reasons. First, because of the south East Asian financial market dynamic character this doesn’t cease attracting all kinds of investors and speculators. Also, the Indonesian warrant market recorded the highest growth rate, in term of stock exchange capitalization, compared to other Indonesian financial markets, particularly after the 1997 crisis. At those circumstances, warrants were issued to encourage reinvesting in Indonesian stocks.

We will try to compare the performances of three warrant valuation models using the data observed on the Jakarta Stock Exchange for the years 2001, 2002 and 2003. The models tested are the Black and Scholes model (BS), the Dilution Adjusted Black and Scholes model (DABS) and the modified Square Root Constant Elasticity Variance model (modified SRCEV) inspired from Cox (1975)\(^4\) Constant Elasticity Variance Model (CEV).

This paper will be structured as follows. The first section introduces the BS and DABS models. The second introduces the CEV model and the necessary modifications, in order to make the Cox Model more suitable for warrants valuation. The third section introduces the statistical summaries of the data used through our empirical study. The fourth section compares the BS and DABS valuation performances. The fifth deals about the underlying assets volatility behaviour. The sixth section compares the modified SRCEV and DABS valuation performances. Finally, we conclude by a brief summary of the main results and some suggestion for future research.

I- Valuating warrants with BS and DABS models

1. **DABS model: Galai and Schneller approach (1978)\(^5\)**

Like a conventional European call, the warrant is the right to buy, at the strike price, an underlying asset at an expiration date. However, warrants are issued by companies so that their exercise will lead to an issue of additional shares and, therefore, a dilution of the issuing company equities. We try to emphasize the relation between the issuing firm value and the warrant price.

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3 Warrants have appeared for the first time on the Jakarta Stock Exchange dated July 13, 1995 and were issued by the bank TIARA ASIA.
4 J. Cox (1975). “Notes on option pricing: Constant Elasticity Variance Diffusions”. Working Paper, Stanford University. Through this paper, Cox has established a new model for pricing options assuming that volatility, is no longer constant, but moves inversely to the underlying asset price.
Let consider a firm (A) with 100% of equities and (n) shares. Suppose that this firm will be liquidated at date (T) with a stochastic future value \( V_T \). Each share value, at that date, will be \( S_T = \frac{V_T}{n} \).

Now let consider a second firm (B), identical to the first, that issues (m) warrants at t=0, with strike price K and expiration date T. Each exercised warrant allows its holder to possess a new issuing firm share. So, the conversion ratio, noted \( \lambda \), is equal to 1. If warrants are exercised at expiration date, the firm B value will be higher than the firm (A) one.

\[
V_T^* = V_T + m \cdot K \quad (1)
\]

The share value of the firm (B) at time T, noted \( S_T^* \), will be equal to

\[
S_T^* = \frac{V_T + m \cdot K}{n + m} \quad (2)
\]

The warrant will be exercised at expiration date, only if the underlying stock price exceeds the warrant strike price.

\[
S_T^* > K \quad \Rightarrow \quad \frac{V_T + m \cdot K}{n + m} > K \quad (3)
\]

In other hand, \( S_T = \frac{V_T}{n} \), so

\[
S_T^* = \frac{n \cdot S_T + m \cdot K}{n + m} > K \quad (4)
\]

Now, let try to infer the warrant price using the BS European call pricing formula. Consider a European call with the firm (A) share as an underlying asset. At expiration date, the call value will be equal to \( \operatorname{Max}(0; S_T - K) \). At the same date, the warrant value will be equal to \( \operatorname{Max}(0; S_T^* - K) \).

\[
S_T^* - K = \frac{n \cdot S_T + m \cdot K}{n + m} - K = n \frac{S_T - K}{n + m} \quad (5)
\]

If \( S_T > K \), both the call and the warrant will be exercised, so

\[
W_T = \frac{n}{n + m} \cdot C_T \quad (6)
\]

If \( S_T < K \), the call and the warrant will be abandoned and will have a null value.

We conclude that the warrant and call prices are perfectly correlated. In an efficient market, and with the no arbitrage assumption, 2 financial assets offering 2 perfectly correlated outputs will have their prices correlated with the same coefficient at any time \( t \in [0; T] \). So

\[
W_t = \frac{n}{n + m} \cdot C_t \quad (7)
\]

As we substitute \( C_t \) by the BS formula, the warrant value can be written as

\[
W_t = \frac{n}{n + m} \left[ S_t \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2) \right] \quad (8)
\]
Consider a conversion ratio $\lambda$, the DABS warrant pricing formula as presented Galai and Schneller is

$$ W_t = \frac{n}{n + m} \cdot \left[ \left( S_t + \frac{m \cdot w}{n} \right) \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) \right] $$

(9)

$W_t$: the warrant price at time (t)
$S_t$: the underlying asset price at time (t)
$K$: the warrant strike price
$r$: riskless interest rate obtained by interpolating the rate for the two bonds whose maturities straddle the warrant expiration date.
$\tau$: the warrant time to expiration

$$ d_1 = \frac{\ln \left( \frac{S_t + \frac{m \cdot w}{n}}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau}{\sigma \cdot \sqrt{\tau}} $$
$$ d_2 = d_1 - \sigma \cdot \sqrt{\tau} $$
$m$: number of issued warrants
$n$: number of underlying shares
$w$: the warrant price at the issue date
$\lambda$: Conversion ratio, i.e. for each warrant exercised, $\lambda$ new underlying shares are created.

2. **BS model: Bensoussan, Crouhy and Galai approach (1995)**

This approach assumes that the underlying asset price reflects the dilution effect. The current shareholders anticipate the effect of an eventual dilution at the issue announcement. At that date, there would be a jump in the underlying asset price. That’s why; Bensoussan, Crouhy and Galai consider that there is no need to make modifications to the BS model when it comes to evaluating warrants because the dilution effect is directly included into the underlying asset price. The warrant theoretical price will be

$$ W_t = S_t \cdot N(d_1) - K \cdot e^{-r \cdot \tau} \cdot N(d_2) \quad (10) $$

$W_t$: the warrant price at time (t)
$S_t$: the underlying asset price at time (t)
$K$: the warrant strike price
$r$: riskless interest rate obtained by interpolating the rate for the two T-bills whose maturities straddle the warrant expiration date.

---

4 Indeed, whenever a warrant holder decides to exercise, the issuing company will be required to create $\lambda$ new shares for him.
\( \tau \): the warrant time to expiration

\[
\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau
\]

\[
d_1 = \frac{\ln \left( \frac{S_t}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) \cdot \tau}{\sigma \cdot \sqrt{\tau}}
\]

\[
d_2 = d_1 - \sigma \cdot \sqrt{\tau}
\]

\( N(). \): the cumulative normal distribution

**II- The modified Square Root Constant Elasticity Variance model**

An extremely practical consequence of the underlying asset price lognormality assumption is that the historical variance can be used to predict the future volatility and thus to evaluate options. Unfortunately, empirical had proven that neither the historical nor the implied volatility can give a future variance estimate which is unique and constant over time. Indeed, there are few chances that the historical variance calculated over the five last years is equal to the historical variance calculated over the last six months. There is considerable evidence in the literature indicating that stock returns are heteroscedastic, with a probability distribution showing a negative skewness. Black (1976)\(^8\) writes “I have believed for long time that stock returns are related to volatility changes. When stocks go up, volatilities seem to go down; and when stocks go down, volatilities seem to go up”.

Cox (1975) proposed a call valuation model which assumed that the volatility is related, by a negative and constant relation, to the asset price. The diffusion process characterizing the Cox model takes the form

\[
\frac{dS}{S} = \mu \cdot dt + \delta \cdot S^{\frac{\theta}{2}} \cdot dz
\]  \hspace{1cm} (11)

\( \mu \): the expected asset return rate

\( \delta \cdot S^{\frac{\theta}{2}} \): the instantaneous standard deviation of the return rate asset, with \( \delta \) and \( \theta \) being constants

\( dz \): a Wiener standard process

The elasticity variance \( h_s \) with respect to the stock price is

\[
\frac{\partial \sigma^2}{\partial S} = \left( \frac{S}{\sigma^2} \right) = \frac{\partial \left( \delta^2 \cdot S^{\theta-2} \right)}{\partial S} \cdot \frac{S}{\delta^2 \cdot S^{\theta-2}} = \theta - 2
\]  \hspace{1cm} (12)

It is easily seen that the model CEV will be equivalent to the BS model when \( \theta=2 \) and that the volatility is a decreasing function of \( S \), as reported by Black (1975), if \( \theta < 2 \). Under the differential equation (11) and the set as assumptions in the BS framework\(^9\), Cox (1975) derived the equilibrium price formula of European call option\(^{10} \) for \( \theta < 2 \). We can clearly see that the elasticity \( h_s \) is constant.


\(^9\) Although the variance is no longer stationary, according to the equation (11), it’s still a deterministic (constant) function of the asset price. The Cox model is a one state variable model, which is once again the asset price. Under these circumstances, it’s possible to establish a perfectly covered position using the call and its underlying asset. When applying the no arbitrage assumption, it becomes possible to obtain the call pricing formula. Moreover, this formula can be applied the risk preferences. Indeed, in a neutral risk economy, individuals require no risk premium. All assets have an expected return rate equal to the risk free interest rate.

\(^{10}\) Cox option pricing formula is developed in appendix A.
and negative. That’s why the Cox model is often called the Constant Elasticity Variance (CEV) model. The CEV European call price details are presented in the appendix.

Several empirical studies, Emanuel and Mac Beth (1982), Bates (1995) and Jones (2003), had shown that the CEV model corrects a systematic bias of the BS model which tends to overprice the in-the-money calls and underprice the out-of-the-money ones. However, when the volatility is inversely related to the asset price, high (low) level asset prices must have low (high) volatilities, so the in-the-money (out-of-the money) calls CEV prices will be lower (higher) than the BS ones.

As the warrant life term is generally higher than one year, we should use a valuation model that allows variance to vary through time. For that reason, using CEV model to pricing warrants should be more appropriate than the use of the BS model. To calculate the CEV warrant price, we introduced 2 main modifications to the general formula set up by Cox. First, we fix $\theta = 1$. This choice will considerably simplify the Cox formula. More, several studies, Beckers (1980)\(^\text{11}\), Lauterbach and Schultz (1990)\(^\text{12}\), Hauser and Lauterbach (1997)\(^\text{13}\), provide encouraging estimates of warrants and options prices when keeping $\theta = 1$. The CEV model will be transformed on what’s called “Square Root Constant Elasticity Variance” (SRCEV) model. (See the formula details in appendix). The second modification made to the CEV model is that we integrated the dilution effect as we did for the DABS model.

The modified SRCEV model is

$$W_t = \frac{n}{n + m} \cdot \left[ (S_t + \frac{m}{n} \cdot w) \cdot N[q(4)] - K \cdot e^{rT} \cdot N[q(0)] \right] \tag{13}$$

$q(4)$ and $q(0)$ are developed in appendix B.

**IV- Database**

The database used for this study consists on:

- Warrants prices: the end-of-the day prices for the warrants negotiated on the Jakarta Stock Exchange (JSX) during the period going from 01/02/2001 until the 12/31/2003. We counted 85 call warrants. Theses prices were directly obtained from the JSX communication and public relationship department.

- Information bulletins: also obtained from the JSX communication and public relationship department. These bulletins indicate, for each warrant, the issue date, the exercise price, the expiry date. The conversion ratio is equal to 1 for all the warrants negotiated on the JSX during this period.

- Underlying shares prices: the end-of-the day shares prices. The total number of the observed prices is 29978. However, neither the database, obtained from the JSX communication and

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\(^{12}\) B. Lauterbach and P. Schultz. “Pricing Warrants: An Empirical Study of the Black Scholes Model and Its Alternatives”. Journal of Finance (1990). This paper, considered as the first one that dealt, in an exhaustive manner, the issue of warrant pricing using the DABS and the SRCEV models.

\(^{13}\) S. Hauser and B. Lauterbach (1997). “The Relative Performance of Five Alternative Warrant Pricing Models”. Financial Analysts Journal. This paper was interested in comparing the performance of five pricing warrants models traded at the American market for the period 1971 to 1980. The models used were the BS model, the DABS model, the Longstaff model for extended warrants, the SRCEV model and the Ritchken free theta model. The latter gave the lowest estimation error (3.6%) ahead of the SRCEV model (3.67%) ranked in 2\(^{nd}\) position.
public relationship department, nor that downloaded from the web site www.jsx.co.id provides information about dividend distribution.

- Riskless Interest Rates: the interest rates for governmental obligations issued during 2001, 2002 and 2003. The information were downloaded from Indonesian Central Bank web site: www.bi.go.id.

Several filters were used in order to reach a sample that can be used for a reliable analysis. We eliminate warrants with incomplete information bulletin. Then, we eliminate warrants inappropriate for the pricing models used through our paper, i.e. the warrants with a “set up” exercise price following a stock split operation. We also apply a liquidity filter. We retain warrants with a liquidity level higher than 20%. The warrants prices that do not match the no arbitrage assumption are excluded from the sample. Finally, we keep the warrants that have, at least, 100 useable observations. The summarized statistics for the final sample are presented at table 1. The distribution of the observation according to the criteria of moneyness\(^{14}\) and time to expiration are presented at table 2. It’s noted that the final sample is dominated by deep out and out-of-the-money warrants, which account for 83.61% of the total observations. Regarding the time to expiration, 73.06% of the observations show a time to expiration beyond a year versus 23.7% with a time to expiration less than a year.

V- “Dilution Adjusted Black & Scholes” vs. “Black & Scholes”

We calculate the estimation error as the absolute deviation of the market warrant price from the theoretical price, reported to the warrant price. This measure illustrates the exactness with which each model fits the observed prices. The average estimation error (AEE), for the whole observations, can be presented as follows

$$AEE = \frac{1}{n} \cdot \sum_{j=1}^{n} \left| \frac{W_{j\text{model}} - W_{j\text{market}}}{W_{j\text{market}}} \right|$$

(14)

Where (n) is the number of observed warrant prices across all warrants and days of the sample.

The AEE, calculated through 5460 observations, are 14.45% and 12.76% respectively for BS and DABS models. Applying the z-test, the minimum significant difference test, we find that the 2 AEE are significantly different from each other at level \(\alpha=1\%\).

Now, let calculate the AEE by warrant. Table (3) provides the results. It also provides the factors that may explain the performance of each of the 2 models.

Except for the warrant TMPO-W, the DABS model provides better estimates than BS model. This superiority can be explained by the fact that DABS model succeeded to correctly treat the dilution effect by making an explicit adjustment of the BS model, via the introduction of the dilution factor \(\frac{n}{n + m}\). An implicit adjustment for the dilution effect through the underlying asset price, as assumed by Bensoussan, Crouhy and Galai (1995), doesn’t seem very suitable for the Indonesian market. We clearly see that the highest difference, between the estimation errors of each model, was

\(^{14}\) Moneyness is calculated as \(\frac{S_t - K \cdot e^{-rt}}{K \cdot e^{-rt}}\) where \(S_t\) is the underlying asset price, \(K\) the warrant exercise price, \(r\) the riskless interest rate and \(\tau\) the time to expiration.
recorded for the warrant that has the highest dilution factor (warrant PLAS-W with a dilution factor of 74%). On the other side, the smallest difference was recorded for the warrant with the smallest dilution factor (warrant ITTG-W with a dilution factor of 9%).

We also classify the observation into 9 subsamples using the warrant moneyness and time to expiration criteria. We made same calculus and same comparison between the 2 models for each of these categories. Results are summarized in table 4.

We clearly see that the DABS average estimation error get closer to the BS one as we move from out-of-the money warrants subsamples to in-the-money ones. Moreover, when applying the z-test, we find that the DABS estimation errors are significantly different from the BS ones (significance at level $\alpha=5\%$) only for deep out-of-the money subsamples. Such a result can be explained by the fact that these warrants are precisely those who have the highest dilution factor (see table 1).

Another finding is that, both DABS and BS models perform the worst for out and deep-out-of-the money warrants, exactly as do the classic BS option valuation formula. This is due to the fact that these 2 models still use the underlying asset price lognormality assumption with a constant volatility. Such an assumption is hardly suitable for a long life term asset as the warrant is.

**VI- the volatility Behaviour**

As shown in table 4, the AEE varies, dramatically, as we move cross subsamples. This may be the result of strong relationship between the local volatility rate and the asset price. To confirm this assumption, we can regress the implied volatility on the moneyness factor. The existence of a significant negative relation between the asset price and its volatility will, necessarily, reinforce the position of the modified SRCEV model versus the BS and DABS ones. To have a more clear idea on this issue, we apply the following regression:

$$ISD_t = \alpha_0 + \alpha_1 \cdot \left( \frac{S_t - K \cdot e^{-rT}}{K \cdot e^{-rT}} \right) + \varepsilon_t$$

(15)

Where $ISD_t$ is the warrant implied standard deviation calculated at date $(t)$, $\left( \frac{S_t - K \cdot e^{-rT}}{K \cdot e^{-rT}} \right)$ is the moneyness factor.

First of all, we do the regression for the whole panel data, across all warrants. We apply three different panel regression models: Fixed Effects Model, Weighted Fixed Effects Model and Random Effects Model. The regression results are summarized at table 5.
Table 1 - DESCRIPTIVE STATISTICS

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Presence in the sample</th>
<th>Obs.¹</th>
<th>Dilution</th>
<th>Expiration</th>
<th>Strike Price</th>
<th>Mean « Moneyness »</th>
<th>Mean ISD²</th>
<th>Mean time to expiration</th>
<th>Life Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>10/08/01 – 12/30/03</td>
<td>385</td>
<td>28.57%</td>
<td>07/19/2004</td>
<td>300</td>
<td>-0.363</td>
<td>0.4759</td>
<td>1.487</td>
<td>3 years</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>01/18/02 – 12/30/03</td>
<td>317</td>
<td>33.33%</td>
<td>01/18/2005</td>
<td>250</td>
<td>-0.261</td>
<td>0.9276</td>
<td>1.759</td>
<td>3 years</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>06/12/01 – 12/30/03</td>
<td>397</td>
<td>12.5%</td>
<td>06/07/2004</td>
<td>250</td>
<td>-0.02</td>
<td>0.3668</td>
<td>1.38</td>
<td>3 years</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>04/16/02 – 12/31/03</td>
<td>380</td>
<td>44.44%</td>
<td>11/22/2004</td>
<td>125</td>
<td>-0.894</td>
<td>1.3365</td>
<td>1.83</td>
<td>3 years</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>08/12/02 – 12/30/03</td>
<td>319</td>
<td>20%</td>
<td>08/11/2005</td>
<td>275</td>
<td>-0.455</td>
<td>0.5059</td>
<td>2.314</td>
<td>3 years</td>
</tr>
<tr>
<td>IDSR-W</td>
<td>03/22/01 – 12/30/03</td>
<td>602</td>
<td>16.67%</td>
<td>03/21/2004</td>
<td>650</td>
<td>0.169</td>
<td>0.9304</td>
<td>1.457</td>
<td>3 years</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>11/26/01 – 12/31/03</td>
<td>490</td>
<td>9.09%</td>
<td>11/25/2004</td>
<td>150</td>
<td>-0.324</td>
<td>0.4829</td>
<td>1.964</td>
<td>3 years</td>
</tr>
<tr>
<td>KARK-W</td>
<td>07/20/01 – 12/30/03</td>
<td>575</td>
<td>48.45%</td>
<td>07/19/2004</td>
<td>125</td>
<td>-0.704</td>
<td>0.819</td>
<td>1.799</td>
<td>3 years</td>
</tr>
<tr>
<td>KREN-W</td>
<td>06/28/02 – 12/30/03</td>
<td>347</td>
<td>16.67%</td>
<td>06/28/2004</td>
<td>265</td>
<td>-0.577</td>
<td>0.5507</td>
<td>2.258</td>
<td>3 years</td>
</tr>
<tr>
<td>META-W</td>
<td>07/18/01 – 07/10/02</td>
<td>239</td>
<td>50%</td>
<td>07/17/2002</td>
<td>200</td>
<td>-0.481</td>
<td>1.139</td>
<td>0.503</td>
<td>1 year</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>03/16/01 – 12/30/03</td>
<td>585</td>
<td>74.07%</td>
<td>03/15/2004</td>
<td>200</td>
<td>-0.538</td>
<td>0.6317</td>
<td>1.473</td>
<td>3 years</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>01/08/02 – 12/31/03</td>
<td>451</td>
<td>44.44%</td>
<td>01/07/2004</td>
<td>300</td>
<td>-0.394</td>
<td>0.7342</td>
<td>1.06</td>
<td>2 years</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>01/02/02 – 12/30/03</td>
<td>466</td>
<td>20%</td>
<td>06/21/2004</td>
<td>175</td>
<td>-0.754</td>
<td>1.1501</td>
<td>1.492</td>
<td>2.5 years</td>
</tr>
</tbody>
</table>

¹The number of observations of each warrant in the sample.
²The implied standard deviation.
Table 2 - Distribution of the Observations according to the moneyness and the time to expiration criteria

The distribution of the 5460 observations will be based on 2 criteria: the moneyness and the time to expiration. Figures in parentheses are the percentages of observations of each subsample compared to the whole sample. Deep in-the-money warrants are those with a moneyness factor > 0.5. In-the-money warrants have a moneyness factor ∈ [0.05; 0.5]. At-the-money warrants have a moneyness ∈ [−0.05; 0.05]. Out-of-the-money warrants have a moneyness factor ∈ [−0.5;−0.05]. Deep out-of-the-money warrants have a moneyness factor <−0.5.

<table>
<thead>
<tr>
<th></th>
<th>≤ 1 year</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0%)</td>
<td></td>
<td>(1.89%)</td>
<td></td>
</tr>
<tr>
<td>Deep In the money</td>
<td>0</td>
<td>(0%)</td>
<td></td>
<td>103</td>
<td>(1.89%)</td>
</tr>
<tr>
<td>In the money</td>
<td>38</td>
<td>(0.70%)</td>
<td></td>
<td>408</td>
<td>(7.47%)</td>
</tr>
<tr>
<td></td>
<td>155</td>
<td>(2.83%)</td>
<td></td>
<td>206</td>
<td>(3.77%)</td>
</tr>
<tr>
<td>At the money</td>
<td>202</td>
<td>(3.70%)</td>
<td></td>
<td>1508</td>
<td>(27.62%)</td>
</tr>
<tr>
<td>Out of the money</td>
<td>899</td>
<td>(16.47%)</td>
<td></td>
<td>1941</td>
<td>(35.55%)</td>
</tr>
</tbody>
</table>

Table 3 - Comparison of Estimation Errors by Warrant

A summary table which aims to compare the performances provided, respectively, by the BS model and the DABS model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors.

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Observations</th>
<th>Mean Moneyness</th>
<th>Dilution</th>
<th>Error BS</th>
<th>Error DABS</th>
<th>Performance BS – DABS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>347</td>
<td>-0.363</td>
<td>28.57%</td>
<td>14.73%</td>
<td>14.28%</td>
<td>0.46%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>299</td>
<td>-0.261</td>
<td>33.33%</td>
<td>3.56%</td>
<td>3.33%</td>
<td>0.23%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>360</td>
<td>-0.02</td>
<td>12.50%</td>
<td>7.15%</td>
<td>5.27%</td>
<td>1.88%</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>379</td>
<td>-0.894</td>
<td>44.44%</td>
<td>21.40%</td>
<td>18.34%</td>
<td>3.06%</td>
</tr>
<tr>
<td>GEMA-W</td>
<td>316</td>
<td>-0.455</td>
<td>20%</td>
<td>13%</td>
<td>12.30%</td>
<td>0.70%</td>
</tr>
<tr>
<td>IDSR-W</td>
<td>601</td>
<td>0.169</td>
<td>16.67%</td>
<td>5.07%</td>
<td>4.91%</td>
<td>0.16%</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>487</td>
<td>-0.324</td>
<td>9.09%</td>
<td>11.93%</td>
<td>11.82%</td>
<td>0.11%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>570</td>
<td>-0.704</td>
<td>48.45%</td>
<td>20.80%</td>
<td>18.82%</td>
<td>1.98%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>344</td>
<td>-0.577</td>
<td>16.67%</td>
<td>15.47%</td>
<td>14.94%</td>
<td>0.53%</td>
</tr>
<tr>
<td>META-W</td>
<td>236</td>
<td>-0.481</td>
<td>50%</td>
<td>24.28%</td>
<td>22.88%</td>
<td>1.40%</td>
</tr>
<tr>
<td></td>
<td>BS</td>
<td>DABS</td>
<td>Difference</td>
<td>Probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>-----</td>
<td>------</td>
<td>------------</td>
<td>-------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deep In + 1 year</strong></td>
<td>4.44%</td>
<td>4.87%</td>
<td>-0.43%</td>
<td>0.2562</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>In + 1 year</strong></td>
<td>5.31%</td>
<td>5.26%</td>
<td>0.05%</td>
<td>0.469</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>In - 1 year</strong></td>
<td>25.58%</td>
<td>35.76%</td>
<td>-10.18%</td>
<td>0.2789</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>At - 1 year</strong></td>
<td>6.86%</td>
<td>5.71%</td>
<td>1.15%</td>
<td>0.2726</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>At + 1 year</strong></td>
<td>10.07%</td>
<td>9.92%</td>
<td>0.15%</td>
<td>0.4859</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Out + 1 year</strong></td>
<td>11.72%</td>
<td>10.71%</td>
<td>1.01%</td>
<td>0.1312</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Out – 1 year</strong></td>
<td>14.55%</td>
<td>12.99%</td>
<td>1.56%</td>
<td>0.293</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deep out - 1 year</strong></td>
<td>16.2%</td>
<td>14.13%</td>
<td>2.07%</td>
<td>0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deep out + 1 year</strong></td>
<td>22.2%</td>
<td>18.31%</td>
<td>3.89%</td>
<td>0.0107</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis probabilities for the z-test.
Table 5 - Comparative Table of Regression Models

Comparative table aimed to identify the estimation results given by each regression model. The criteria used in this comparison are: the calculated (t) student, the regression linearity coefficient and the sum of residual squares. We compare the Fixed Effects Model (FEM), the weighted FEM and the Random Effects Model (REM).

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>FEM</th>
<th>weighted FEM</th>
<th>REM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.455</td>
<td>-1.331</td>
<td>-1.480</td>
</tr>
<tr>
<td>$t(\alpha_1)$</td>
<td>-64.708</td>
<td>-99.205</td>
<td>-83.555</td>
</tr>
<tr>
<td>$R^2$</td>
<td>61.69%</td>
<td>78.21%</td>
<td>70.28%</td>
</tr>
<tr>
<td>$R^2_{\text{adjusted}}$</td>
<td>61.6%</td>
<td>78.15%</td>
<td>70.28%</td>
</tr>
<tr>
<td>SCR</td>
<td>506.673</td>
<td>382.446</td>
<td>393.009</td>
</tr>
</tbody>
</table>

All regression models show a strong inverse relationship between implied volatility and moneyness. The three $\alpha_1$ estimations are significantly negative at level $\alpha=1\%$. The best regression model seems to be the Weighted Fixed Effects Model providing the most significant $\alpha_1$ estimation with the highest “t” student, the best linearity and adjusted linearity coefficients and the lowest sum of squared residuals. Such a result suggests that the modified SRCEV model would be more appropriate to fit warrants prices than DABS and BS models.

According to stochastic volatility models, Wiggins (1987)$^{18}$, Hull and White (1987)$^{19}$, Heston (1993)$^{20}$, the volatility varies through time without being linked to the underlying asset price by a deterministic relation. In other terms, these models allow arbitrary correlation between volatility and asset price. Such volatility behaviour, as shown by Heston (1993), produces a probability distribution of the asset returns with high kurtosis. Thus, the relation between the implied volatility and the underlying asset price can vary as the warrant is in or out-of-the money.

To verify whether the volatility is purely stochastic or not, we repeat the regression, using the weighted Fixed Effect Model, only for in-the-money warrants. The in-the-money observations represent 12% of the whole panel (665 observations among a total of 5460 warrants prices). The regression results are summarized in table 6.

---


Table 6 - Regression results for in-the-money observations

<table>
<thead>
<tr>
<th>Warrant</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>0.1442</td>
<td></td>
</tr>
<tr>
<td>ANTA-W</td>
<td>0.3029</td>
<td></td>
</tr>
<tr>
<td>BCAP-W</td>
<td>0.2636</td>
<td></td>
</tr>
<tr>
<td>IDSR-W</td>
<td>0.6177</td>
<td>-0.1742</td>
</tr>
<tr>
<td>ITTG-W</td>
<td>0.2629</td>
<td></td>
</tr>
<tr>
<td>KARK-W</td>
<td>0.1735</td>
<td></td>
</tr>
<tr>
<td>META-W</td>
<td>0.1753</td>
<td></td>
</tr>
<tr>
<td>PLAS-W</td>
<td>0.1245</td>
<td></td>
</tr>
<tr>
<td>TMPO-W</td>
<td>0.1483</td>
<td></td>
</tr>
</tbody>
</table>

$R^2 = 93.81\%$  $R^2_{\text{adjusted}} = 93.72\%$

Although the inverse relation is more pronounced for out-of-the-money warrants, the regression has shown that such relation is maintained for in-the-money ones with an $\alpha_1$ estimation significantly negative at level $\alpha = 1\%$. Then, we can deduce that the volatility remains inversely related to the underlying asset price, as assumed by the SRCEV model, and not purely stochastic.

VII- Modified “SRCEV” vs. “DABS”

The AEE, calculated through the 5460 observations, are 8.95% and 12.76% respectively for modified SRCEV and DABS models. Such a result was predictable as the modified SRCEV is adapted to the 2 warrant specificities, i.e. the dilution effect and the warrant long life term, by incorporating the dilution factor in the original pricing formula and using a variable volatility. The z-test shows that the 2 AEE are significantly different from each others at level $\alpha = 1\%$. Table 7 compares the AEE calculated for each warrant using the 2 models.

Table 7 - Comparison of Estimation Errors by Warrant

<table>
<thead>
<tr>
<th>Warrant</th>
<th>Average Moneyness</th>
<th>Error DABS</th>
<th>Error SRCEV</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIMS-W</td>
<td>-0.363</td>
<td>14.275%</td>
<td>8.20%</td>
<td>6.075%</td>
</tr>
<tr>
<td>ANTA-W</td>
<td>-0.261</td>
<td>3.33%</td>
<td>0.47%</td>
<td>2.86%</td>
</tr>
<tr>
<td>BCAP-W</td>
<td>-0.02</td>
<td>5.27%</td>
<td>5.27%</td>
<td>0.01%</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>18.34%</td>
<td>14.13%</td>
<td>4.21%</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
<td>--------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>CNKO-W</td>
<td>-0.894</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEMA-W</td>
<td>-0.455</td>
<td>12.3%</td>
<td>8.85%</td>
<td>3.45%</td>
</tr>
<tr>
<td>IDS-R-W</td>
<td>0.169</td>
<td>4.91%</td>
<td>3.49%</td>
<td>1.42%</td>
</tr>
<tr>
<td>ITT-G-W</td>
<td>-0.324</td>
<td>11.82%</td>
<td>11.29%</td>
<td>0.53%</td>
</tr>
<tr>
<td>KARK-W</td>
<td>-0.704</td>
<td>18.82%</td>
<td>9.81%</td>
<td>9.01%</td>
</tr>
<tr>
<td>KREN-W</td>
<td>-0.577</td>
<td>14.94%</td>
<td>10.17%</td>
<td>4.77%</td>
</tr>
<tr>
<td>META-W</td>
<td>-0.481</td>
<td>22.876%</td>
<td>15.31%</td>
<td>7.566%</td>
</tr>
<tr>
<td>PLAS-W</td>
<td>-0.538</td>
<td>12.72%</td>
<td>12.14%</td>
<td>0.58%</td>
</tr>
<tr>
<td>TMPO-W</td>
<td>-0.394</td>
<td>13.89%</td>
<td>9.72%</td>
<td>4.17%</td>
</tr>
<tr>
<td>WAPO-W</td>
<td>-0.754</td>
<td>14.42%</td>
<td>8.59%</td>
<td>5.83%</td>
</tr>
</tbody>
</table>

Now let calculate the AEE across subsamples made upon the moneyness and the time to expiration criteria. Results are summarized at table 8.

We find that the modified SRCEV estimation errors are significantly different, at level $\alpha = 5\%$, from the DABS errors, for the out-of-the money subsamples. On the other hand, we can see that the 2 models estimation errors are not significantly different from each others, for at and in-the-money warrants. The 2 models provide an almost similar performance for these categories of warrants.

Such a result can be explained by the modified SRCEV assumption for fixed $\theta = 1$ across all the moneyness categories and for all warrants. This assumption may be suitable for out-of-the money warrants, where the modified SRCEV model had performed its best performance, but it seems that it doesn’t work so good for at and in-the-money subsamples. For these categories of warrants, there is still an inverse relation between volatility and asset price (as shown in the previous section) but the correlation between these 2 variables should be more moderate than for out-of-the money warrants. Investors tend to be very frightened face any downtrend in the market and to demonstrate a very moderate optimism when the movement becomes bullish. Such behaviour is quite remarkable response to periods of financial crises. The volatility decreases monotonically as the asset price goes upward but the rate of decrease should be less than the rate of volatility increase when asset price goes downward. Thus $\theta$ must vary as we move from out-of-the money to in-the-money subsamples.
Table 8 - Estimation Errors classified by Moneyness and Time to expiration

A summary table which aims to compare the performances provided, respectively, by the DABS model and the SRCEV model. This comparison will be made by using the average estimation error of each model, then by calculating the difference between the 2 average errors. We apply the z-test for each difference. Deep in-the-money warrants are those with a moneyness factor>0.5. In-the-money warrants have a moneyness factor $\in [0.05;0.5]$. At-the-money warrants have a moneyness $\in [-0.05;0.05]$. Out-of-the-money warrants have a moneyness factor $\in [-0.5;\infty)$. Deep out-of-the-money warrants have a moneyness factor<-0.5.

<table>
<thead>
<tr>
<th></th>
<th>DABS</th>
<th>SRCEV</th>
<th>Difference</th>
<th>Probability(^{21})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep In + 1 year</td>
<td>4.87 %</td>
<td>3.98 %</td>
<td>0.89 %</td>
<td>0.318</td>
</tr>
<tr>
<td>In + 1 year</td>
<td>5.26 %</td>
<td>4.36 %</td>
<td>0.9 %</td>
<td>0.411</td>
</tr>
<tr>
<td>In - 1 year</td>
<td>35.76 %</td>
<td>14.76 %</td>
<td>21 %</td>
<td>0.116</td>
</tr>
<tr>
<td>At - 1 year</td>
<td>9.92 %</td>
<td>9.01 %</td>
<td>0.91 %</td>
<td>0.420</td>
</tr>
<tr>
<td>At + 1 year</td>
<td>5.71 %</td>
<td>5.54 %</td>
<td>0.17 %</td>
<td>0.1574</td>
</tr>
<tr>
<td>Out + 1 year</td>
<td>10.71 %</td>
<td>8.26 %</td>
<td>2.45 %</td>
<td>0.0384</td>
</tr>
<tr>
<td>Out – 1 year</td>
<td>12.99 %</td>
<td>11.03 %</td>
<td>1.96 %</td>
<td>0.0005</td>
</tr>
<tr>
<td>Deep out - 1 year</td>
<td>18.31 %</td>
<td>10.89 %</td>
<td>7.42 %</td>
<td>0.0005</td>
</tr>
<tr>
<td>Deep out + 1 year</td>
<td>14.13 %</td>
<td>9.72 %</td>
<td>4.41 %</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Conclusion

As expected, the modified SRCEV model led to the lowest AEE for the whole panel, for each warrant and for the different moneyness subsamples. This result can be explained by the fact that modified SRCEV is the only one of the 3 models used in our empirical study, which tries to exploit the 2 major warrant specificities.

First, the modified SRCEV is explicitly adjusted for the dilution effect which should allow offsetting the negative impact that dilution could have on the quality of estimation. Also, unlike BS and DABS models, the modified SRCEV is more adapted to the warrant long life term, proposing a diffusion process with variable underlying asset price volatility. This assumption seems to be more consistent than that of constant volatility used by BS and DABS models.

However, the modified SRCEV model may be subject to some criticism. We can refer, in particular, its use of a constant elasticity variance across all the underlying assets. This elasticity can even vary for the same underlying asset depending if the spot price is going upward or downward.

To remedy this deficiency, one can calculate the implied elasticity using the observed warrants prices and then calibrate the modified SRCEV model so it can be adapted for the underlying asset specificities.

\(^{21}\) The null hypothesis probabilities for the z-test.
Bibliography


Appendix A - Cox Option Pricing Formula

Cox option pricing formula under the assumption of a constant elasticity variance is defined in the following manner:

\[ C_v = \sum_{n=0}^{\infty} g(\lambda \cdot S^\phi, n+1) \cdot G\left( \lambda \cdot (K \cdot e^{-\tau r})^\phi, n+1 - \frac{1}{\phi} \right) - K \cdot e^{-\tau r} \cdot \sum_{n=0}^{\infty} g(\lambda \cdot S^\phi, n+1 - \frac{1}{\phi}) \cdot G\left( \lambda \cdot (K \cdot e^{-\tau r})^\phi, n+1 \right) \]

\( C_v \) is a European call price with a strike price K and a time to expiration \( \tau = (T - t) \);
\( \phi = 2 \theta - 2 \);
\( \lambda = 2 \frac{r}{\delta^2} \cdot \phi \cdot e^{(\theta \cdot r - 1)} \);
\( \Gamma(n) = \int_0^\alpha e^{-v} \cdot v^{n-1} \cdot dv \) : the gamma function;
\( g(x, n) = e^{-x} \cdot x^{n-1} \cdot \frac{1}{\Gamma(n)} \) : the density gamma function;
\( G(a, n) = \int_0^\alpha g(x, n) \cdot dx \) : the complementary gamma distribution function.

Appendix B - SRCEV option pricing formula as proposed by Beckers (1980)

For \( v = 0 \) or \( v = 4 \),
\( (v) = \frac{1 + h(h - 1) \cdot p - \frac{1}{2} \cdot h(h - 1)(2 - h)(1 - 3h) p^2 - \frac{z}{(v + y)^q}}{[2h^2 \cdot p \cdot [1 - (1 - h)(1 - 3h)p]]^{0.5}}, \)
\( h(v) = 1 - \frac{2(v + y) \cdot (v + 3y)}{3(v + 2y)^2}, \) \( p(v) = \frac{(v + 2y)}{(v + y)^2}, \) \( y = \frac{4r \cdot (S + \frac{M \cdot N - W}{N})}{\sigma^2 (1 - e^{-\tau r})} \) et \( z = -\frac{K \cdot y}{S + \frac{M \cdot N}{N} \cdot W}. \)